9/24/14

4. Homework

Due **10/3/14** at the beginning of class

Remember, you are allowed to turn in homeworks in groups of two. One writeup, with two names.

1. **Injective, Surjective, Bijective (6 points)**

   (a) $f : \mathbb{R} \to \mathbb{R}; f(x) = -2x + 5$

   (b) $f : \mathbb{R} \to \mathbb{R}; f(x) = -2x^2 + 6$

   (c) $f : \mathbb{R} \to \mathbb{R}; f(x) = (x + 1)/(x + 2)$

   For each of the above functions, perform the following three tasks:

   (i) Determine whether it is a valid function. If not, change the domain to make it a valid function.

   (ii) Determine whether it is injective, surjective, or bijective.

   (iii) If the function is not surjective, please specify how to alter the codomain to make it surjective.

2. **One-to-one and Onto (2 points)**
   Give an example of a function from $\mathbb{N}$ to $\mathbb{N}$ that is:

   (a) onto but not one-to-one.

   (b) neither onto nor one-to-one.

   Justify your answer.

3. **Composition (5 points)**

   (a) Find $f \circ g$ and $g \circ f$ where $f(x) = x^2 + 2$ and $g(x) = x + 1$ are functions from $\mathbb{R}$ to $\mathbb{R}$.

   (b) Let $f(x) = ax + b$ and $g(x) = cx + d$, where a, b, c, d are constants. Determine for which constants a, b, c, d it is true that $f \circ g = g \circ f$.

4. **Composition of Functions (6 points)**
   Prove that the composition of two injective functions is injective. Proceed in the following steps:

   - Formalize the theorem statement in a logical formula that uses three functions f, g, h.
   - Draw a picture that illustrates the claim.
   - Now apply the definition of an injective function to prove the claim.

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5. **Sequences (2 points)**
   Find two different sequences beginning with the terms $3, 5, 7$ whose terms are generated by a simple formula or by an (informally described) rule.

6. **Summations (3 points)**
   Find explicit formulas for the following summations:

   (a) $\sum_{i=1}^{10}(2i+1)$

   (b) $\sum_{i=1}^{n}(2i+1)$

   (c) $\sum_{i=4}^{n} 2^{i-4}$