CMPS/MATH 2170 Discrete Mathematics – Fall 14

9/17/14

3. Homework

Due **9/26/14** at the beginning of class

Remember, you are allowed to turn in homeworks in groups of two. One writeup, with two names.

1. **Equivalence (3 points)**
   Prove that the following are equivalent for all $a, b \in \mathbb{R}$:
   (i) a is less than b, (ii) the average of a and b is greater than a, (iii) the average of a and b is less than b

2. **Irrational (4 points)**
   (a) Prove or disprove: If x and y are rational, then x^y is rational.
   (b) Prove that $\sqrt{2} - 1$ is irrational. What proof strategy did you use?

3. **Indirect Proofs (6 points)**
   Consider the following statement:
   Let n be an integer. If $n^2 - 3n - 5$ is even, then n is odd.
   (a) Prove the statement using an indirect proof (by contrapositive).
   (b) Prove the statement using a proof by contradiction.
   (c) Likely both your proofs are very similar and shared some of the same arguments. Can you formally state why a proof by contradiction and an indirect proof by contrapositive can be similar at times?

4. **Sets (11 points)**
   (a) (3 points) Which of these statements are true? Justify your answers.
      i. $\emptyset \in \{\emptyset\}$
      ii. $\{\emptyset\} \in \{\emptyset\}$
      iii. $\{\emptyset\} \subseteq \{\emptyset, \{\emptyset\}\}$
   (b) (2 points) Let $A_i = \{i, i+1, i+2, \ldots\}$.
      i. Give a concise description for $\bigcup_{i=1}^{\infty} A_i$.
      ii. Give a concise description for $\bigcap_{i=1}^{\infty} A_i$.
   (c) (2 points) Show that if A is a subset of B, then the power set of A is a subset of the power set of B.
   (d) (1 point) Show that if A and B are sets, then $(A \cup B) \subseteq (A \cup B \cup C)$.
   (e) (3 points) Show that if A and B are sets, then $(A \cap B) \cup (A \cap \bar{B}) = A$.