Triangulations

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Triangulations

- Decompose the polygon into shapes that are easier to handle: triangles
- A **triangulation** of a polygon $P$ is a decomposition of $P$ into triangles whose vertices are vertices of $P$. In other words, a triangulation is a maximal set of non-crossing diagonals.
Triangulations

- A polygon can be triangulated in many different ways.
Triangulations of Simple Polygons

**Theorem 1:** Every simple polygon admits a triangulation, and any triangulation of a simple polygon with $n$ vertices consists of exactly $n-2$ triangles.

**Proof:** By induction.

- $n=3$: △
- $n>3$: Let $u$ be leftmost vertex, and $v$ and $w$ adjacent to $v$. If $vw$ does not intersect boundary of $P$: \#triangles $= 1$ for new triangle + $(n-1)-2$ for remaining polygon $= n-2$
**Triangulations of Simple Polygons**

**Theorem 1:** Every simple polygon admits a triangulation, and any triangulation of a simple polygon with \( n \) vertices consists of exactly \( n-2 \) triangles.

If \( vw \) intersects boundary of \( P \): Let \( u' \neq u \) be the vertex furthest to the left of \( vw \). Take \( uu' \) as diagonal, which splits \( P \) into \( P_1 \) and \( P_2 \).

\[
\text{#triangles in } P = \text{#triangles in } P_1 + \text{#triangles in } P_2 = |P_1|-2 + |P_2|-2 = |P_1|+|P_2|-4 = n+2-4 = n-2
\]
Triangulate an $l$-Monotone Polygon

- Using a greedy plane sweep in direction $l$
- Sort vertices by increasing $x$-coordinate (merging the upper and lower chains in $O(n)$ time)
- Greedy: Triangulate everything you can to the left of the sweep line.