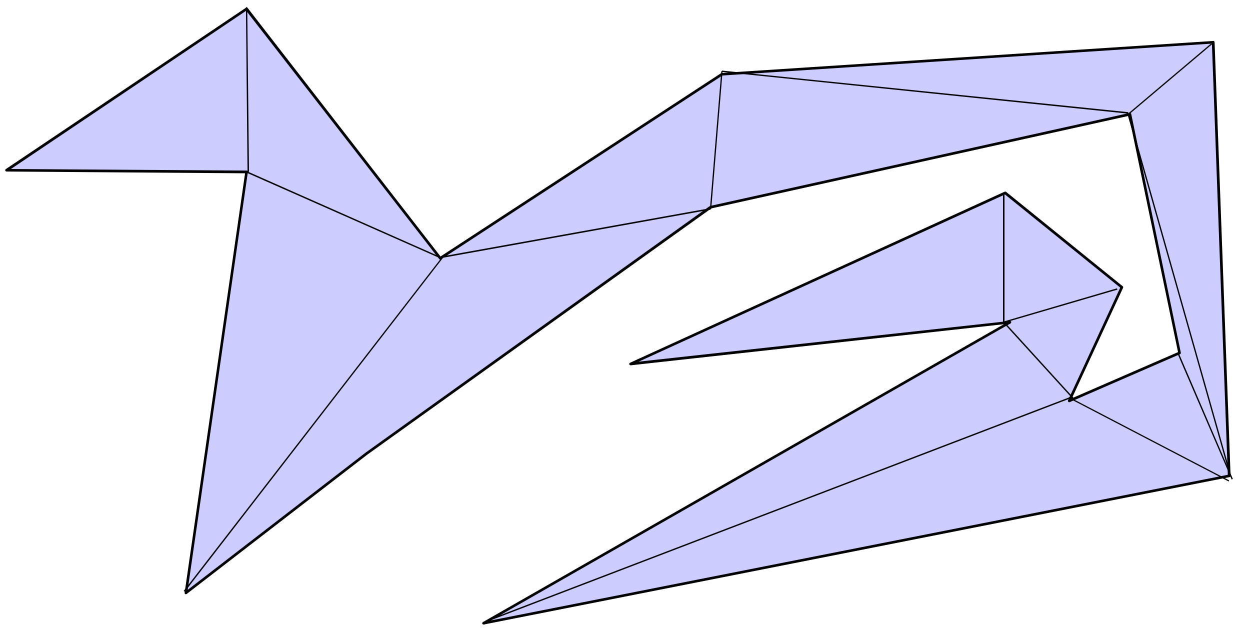
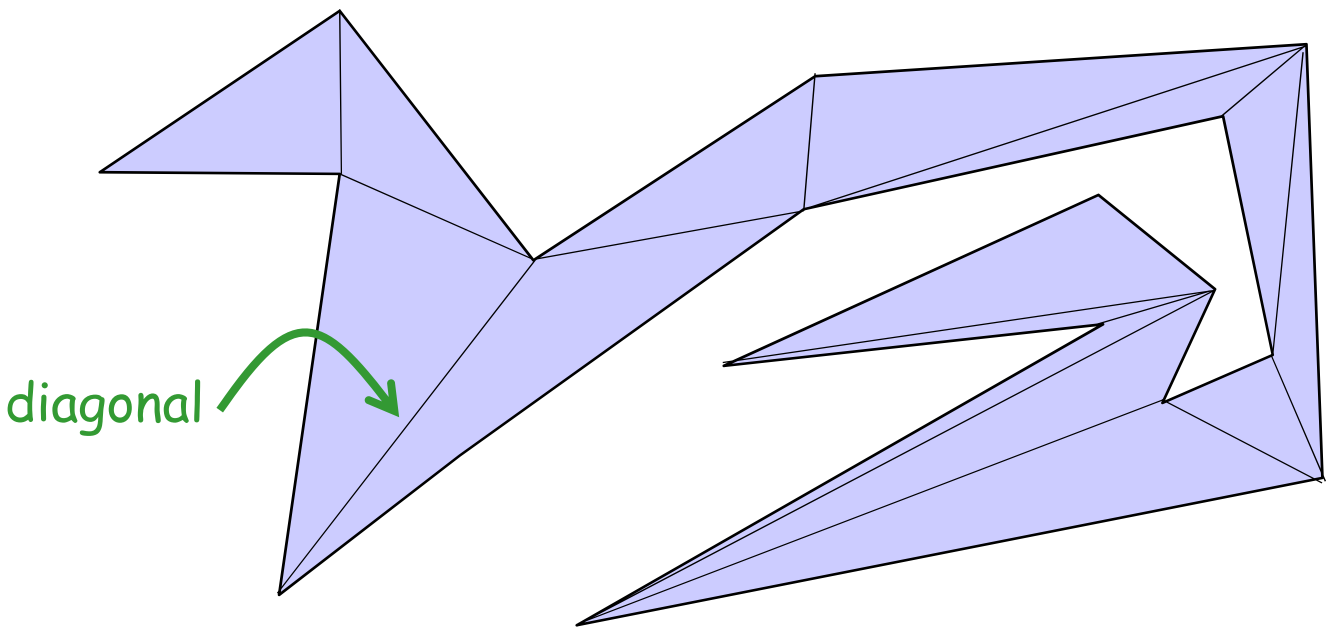


Triangulations

Carola Wenk

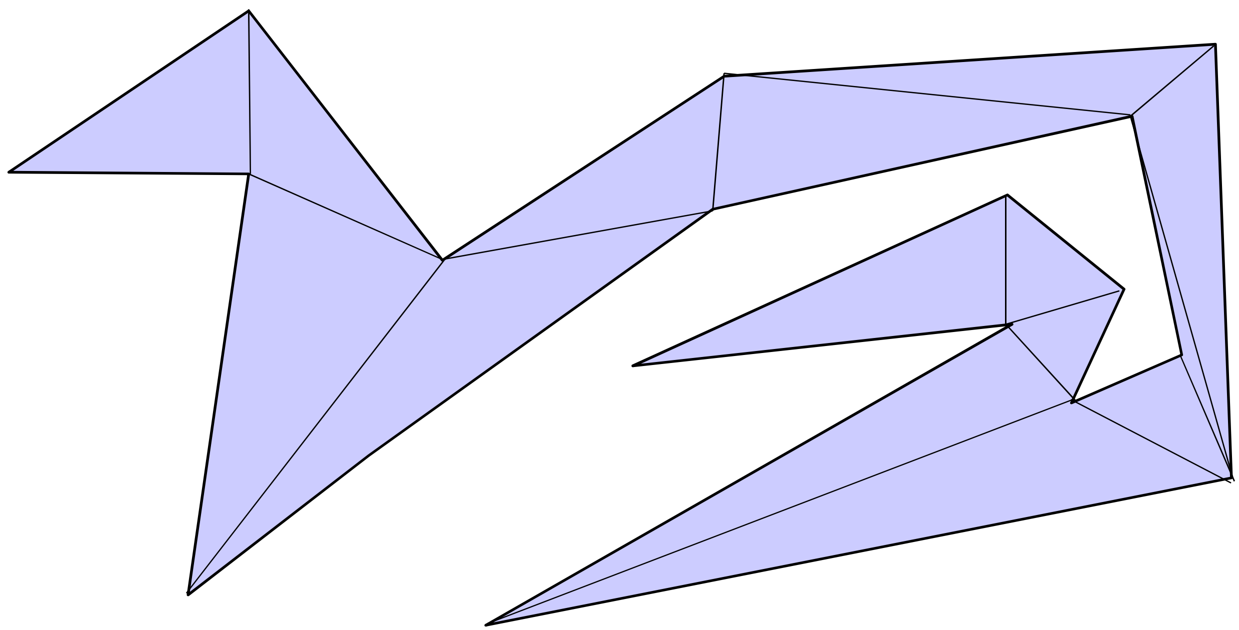
Triangulations

- Decompose the polygon into shapes that are easier to handle: triangles
- A **triangulation** of a polygon *P* is a decomposition of *P* into triangles whose vertices are vertices of *P*. In other words, a triangulation is a maximal set of non-crossing diagonals.



Triangulations

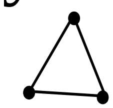
- A polygon can be triangulated in many different ways.

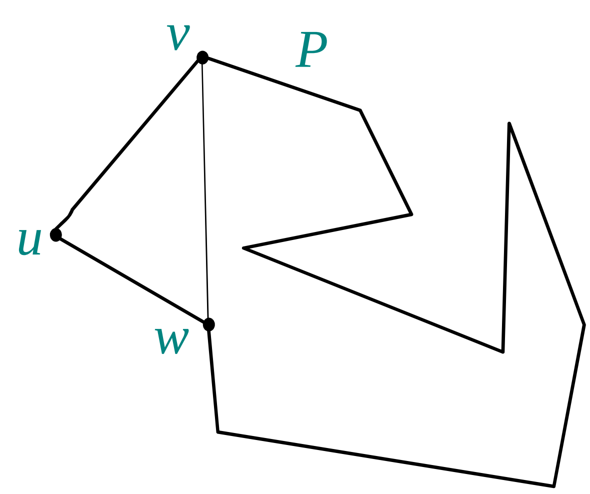


Triangulations of Simple Polygons

Theorem 1: Every simple polygon admits a triangulation, and any triangulation of a simple polygon with n vertices consists of exactly n-2 triangles.

Proof: By induction.

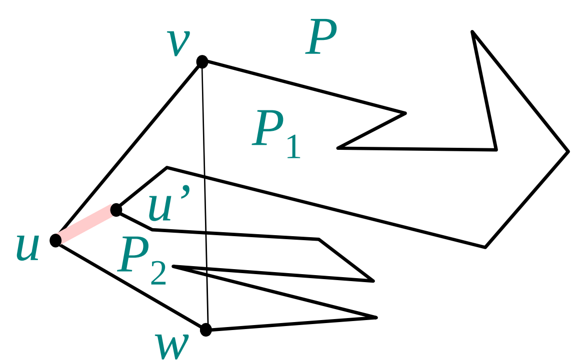
- n=3: 
- n>3: Let u be leftmost vertex, and v and w adjacent to v. If $\overline{vw}$ does not intersect boundary of P: #triangles = 1 for new triangle + (n-1)-2 for remaining polygon = n-2



Triangulations of Simple Polygons

Theorem 1: Every simple polygon admits a triangulation, and any triangulation of a simple polygon with n vertices consists of exactly n-2 triangles.

If $\overline{vw}$ intersects boundary of P: Let $u' \neq u$ be the the vertex furthest to the left of $\overline{vw}$. Take $\overline{uu'}$ as diagonal, which splits P into P_1 and P_2.
#triangles in P = #triangles in P_1 + #triangles in P_2 = $|P_1|-2 + |P_2|-2 = |P_1|+|P_2|-4 = n+2-4 = n-2$



Triangulate an *l*-Monotone Polygon

- Using a greedy plane sweep in direction *l*
- Sort vertices by increasing *x*-coordinate (merging the upper and lower chains in O(*n*) time)
- Greedy: Triangulate everything you can to the left of the sweep line.