CMPS/MATH 2170 Discrete Mathematics – Fall 13

11/25/13

# Extra Credit Homework

Due **Friday 12/6/13** at the beginning of class

1. **Relations (11 points)**
   (a) (2 points) Determine whether the following relation $R$ on the set of all people is reflexive, symmetric, antisymmetric, and/or transitive, where $(a, b) \in R$ iff $a$ and $b$ were born on the same day.
   (b) (1 point) Give an example of a relation on a set that is reflexive and not symmetric. Justify your answer.
   (c) (4 points) Let $R$ be a relation on a set. The *inverse relation* $R^{-1}$ is defined as $R^{-1} = \{(b, a) \mid (a, b) \in R\}$. Show that $R$ is antisymmetric if and only if $R \cap R^{-1}$ is a subset of the diagonal relation $\Delta = \{(a, a) \mid a \in A\}$.
   (d) (4 points) Let $R$ be the relation on the set of all sets of real numbers such that $(S, T) \in R$ if and only if $S$ and $T$ have the same cardinality. Show that $R$ is an equivalence relation. What are the equivalence classes of the sets $\{0, 1, 2\}$ and $\mathbb{Z}$?

2. **Graphs (4 points)**
   (a) (2 points) Let $G = (V, E)$ be a (simple and undirected) graph. Let $B$ be the maximum degree of all vertices, and let $A$ be the minimum degree of all vertices. Show that $A \leq 2|E|/|V| \leq B$.
   (b) (2 points) Describe the adjacency matrix of a graph with $k$ connected components when the vertices of the graph are listed so that vertices in each connected component are listed successively.

3. **Trees (6 points)**
   (a) (4 points) Let $G$ be a simple graph. Show that $G$ is a tree if and only if (i) G is connected and (ii) the deletion of any of its edges produces a graph that is not connected.
   *(Hint: Show $G$ is a tree implies (i) and (ii). Then show (i) and (ii) imply that $G$ is a tree.)*
   (b) (2 points) Show that a connected graph with $n$ vertices has to have at least $n - 1$ edges.