11/4/13

9. Homework

Due **11/13/13** at the beginning of class

1. **Pigeonholes (6 points)**

   (a) (2 points) There are 51 houses on a street. Each house has a house number between 1000 and 1099, inclusive. Show that at least two houses have house numbers that are consecutive integers.

   (b) (2 points) How many students must minimally be enrolled at Tulane to ensure that at least 100 students are from the same state (assuming there are 50 states in the US)?

   (c) (2 points) Show that if five points are picked in the interior of a square with side length of 2, then at least two of these points are no farther than $\sqrt{2}$ apart.
   (Hint: $\sqrt{2}$ is the length of the diagonal of a square with side length 1.)

2. **Permutations and Combinations (4 points)**
   Let $S = \{1, 2, 3, 4, 5\}$ be a set of $n = 5$ elements, and let $r = 3$. Give a description of the items that are being counted, as well as their number, for each of the following scenarios:

   (a) r-permutations of S without repetition

   (b) r-combinations of S without repetition

   (c) r-permutations with repetition

   (d) r-combinations with repetition

   (Hint: Consider the overview table for permutations and combinations that we covered in class.)

3. **String Counting (4 points)**

   (a) (2 points) How many permutations of the letters ABCDEFGH contain the string BEG?

   (b) (2 point) How many different strings can be made from the word PEPPERCORN when all the letters are used?

4. **More Counting (4 points)**
   How many different ways are there to choose a dozen donuts from the 21 varieties at a donut shop...

   (a) (2 points) ... such that no two donuts are of the same variety?

   (b) (2 points) ... if there are no restrictions? (I.e., now it is allowed to choose multiple donuts from one flavor; one could even choose all donuts from one flavor.)

5. **Subsets and Binomial Coefficients (4 points)**
   Prove that $\binom{2n}{2} = 2\binom{n}{2} + n^2$, for any $n \in N$, ...

   (a) (2 points) ... using a combinatorial argument.

   (b) (2 points) ... by algebraic manipulation.