CMPS/MATH 2170 Discrete Mathematics – Fall 13

10/21/13

7. Homework

Due **10/30/13** at the beginning of class

1. **GCD (8 points)**
   (a) (2 point) Find $gcd(1000, 625)$ and $lcm(1000, 625)$ and verify that $gcd(1000, 625) \cdot lcm(1000, 625) = 1000 \cdot 625$.
   (b) (2 points) Use the Euclidean algorithm to find $gcd(277, 123)$.
   (c) (4 points) Prove $a \equiv b \pmod{m}$ if and only if $a \bmod m = b \bmod m$, for all $a, b \in \mathbb{Z}$ and $m \in \mathbb{N}$.

2. **Euler's Φ (5 points)**
   Euler's Φ-function is defined as follows for any $n \in \mathbb{N}$: $\Phi(n)$ is the number of positive integers less than or equal to n that are relatively prime to n.
   (a) (1 point) Find $\Phi(6)$ and $\Phi(7)$
   (b) (4 points) Prove that n is prime if and only if $\Phi(n) = n - 1$.

3. **Inverse (6 points)**
   (a) (1 point) Find an inverse of 2 modulo 17.
   (b) Theorem: Show that an inverse of a modulo m does not exist if $gcd(a, m) > 1$.
      i. (2 points) For $m = 10$ and $a = 6$, what is $gcd(a, m)$? Argue why an inverse of $a = 6$ does not exist.
      ii. (3 points) Prove the theorem statement.

4. **Perfect Number (5 points)**
   A positive integer is called *perfect* if it equals the sum of its positive divisors other than itself.
   (a) (2 point) Show that 6 and 28 are perfect.
   (b) (3 points) Show that $2^{p-1}(2^p - 1)$ is a perfect number, when $2^p - 1$ is prime. *(Hint: Use the formula for the geometric series.)*