CMPS/MATH 2170 Discrete Mathematics – Fall 13

10/14/13

6. Homework

Due **10/23/13** at the beginning of class

1. **Recursive sequence (4 points)**
   Give a recursive definition of the sequence $\{a_n\}_{n \in \mathbb{N}}$ if
   (a) (2 point) $a_n = 4n - 2$
   (b) (2 point) $a_n = n(n+1)$

2. **Recursive Definition (4 points)**
   Give a recursive definition of:
   (a) (2 point) the set of odd positive integers.
   (b) (2 point) the set of positive integers that are multiples of 3.

3. **Fibonacci (6 points)**
   Let F_n be the n-th Fibonacci number ($F_0 = 0, F_1 = 1, F_n = F_{n-1} + F_{n-2}$ for all $n \geq 2$). Let s and t be constants. And let $\{a_n\}_{n \in \mathbb{N}}$ be defined as $a_0 = s$, $a_1 = t$, and $a_n = a_{n-1} + a_{n-2}$ for all $n \geq 2$. Use induction to show that for all $n \in \mathbb{N}$:

   $$a_n = sF_{n-1} + tF_n$$

   (Hint: $\mathbb{N} = \{1, 2, 3, \ldots\}$*. And make sure that you have two base cases, since the recursive definition for* a_n *refers to two previous values.)*

4. **Expansion and induction (4 points)**
   Consider the recursively defined function $T(0) = a$ and $T(n) = T(n-1) + b$ for all $n \geq 1$.
   (a) (1 point) Apply the expansion method to arrive at a guess what $T(n)$ might solve to. Show your work.
   (b) (3 points) Prove by induction that $T(n) = nb + a$ for all $n \in \mathbb{N}_0$.

5. **Climbing a ladder (5 points)**
   Consider climbing a ladder with n rungs. The rungs are spaced such that you can climb one rung, two rungs, or three rungs at a time. Let $r(n)$ be the number of different ways to climb a ladder with n rungs. For example, $r(2) = 2$ because one can climb a 2-rung ladder either as $1 + 1$ rungs or as 2 rungs, which are two different climbing patterns.
   (a) (1 point) Give the values of $r(1), r(3), r(4)$; justify your answers.
   (b) (3 point) Develop a recursive formula for $r(n)$. Explain your answer. *(Hint: This will look similar to the Fibonacci numbers, with multiple base cases and more than one recursive "call".*
   (c) (1 point) What is $r(7)$?