CMPS/MATH 2170 Discrete Mathematics – Fall 13

9/23/13

4. Homework

Due **10/2/13** at the beginning of class

1. **Functions (8 points)**
   Justify all your answers.

   (a) (1 point) Give the mathematical notation (including domain and co-domain) of the function that assigns to each pair of positive integers the first integer of the pair.

   (b) (1 point) Let $f(x) = ax + b$ and $g(x) = cx + d$. Determine for which constants a, b, c, d it is true that $f \circ g = g \circ f$.

   (c) (2 points) Determine whether $f(x) = -3x^2 + 7$ is a bijection from $\mathbb{R}$ to $\mathbb{R}$. If it is not a bijection, can you specify a different domain or co-domain for which f is bijective?

   (d) (2 points) Determine whether $f(x) = 4x^3$ is a bijection from $\mathbb{R}$ to $\mathbb{R}$. If it is not a bijection, can you specify a different domain or co-domain for which f is bijective?

   (e) (2 points) Let $f : B \to C$ be bijective and let $g : A \to B$ be surjective. Prove that $f \circ g$ is surjective. Does this still hold if f is only surjective?

2. **Functions II (3 points)**
   Give an example of a function from $\mathbb{N}$ to $\mathbb{N}$ that is:

   (a) one-to-one but not onto

   (b) onto but not one-to-one

   (c) neither one-to-one nor onto

   Justify your answers.

3. **Sequences (3 points)**
   For each of the sequences below, find a formula that generates the sequence.

   (a) $7, 11, 15, 19, 23, 27, 31, \ldots$

   (b) $5, 15, 45, 135, 405, \ldots$

   (c) $3, 6, 11, 18, 27, 38, 51, \ldots$

4. **Summation (2 points)**
   Find an explicit formula for the summation below. Use index substitution. Simply the formula as much as possible.

$$\sum_{i=3}^{n} \left(\frac{1}{2}\right)^{i-3}$$

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5. **Cardinality (5 points)**
   Determine whether the sets below are finite, countably infinite, or uncountable.

   (a) (1 point) The negative integers.

   (b) (2 points) The real numbers between 0 and 1/2, inclusive.

   (c) (2 points) The real numbers with decimal representations consisting of all 1s.

6. **More Cardinality (2 points)**
   Let A be a countable set. Show that the set B is also countable if there is a surjective function from A to B.