

3. Homework

Due **9/25/13** at the beginning of class

1. Contrapositive and contradiction (4 points)

Consider the following claim:

For all integers m, n , if mn is even then m is even or n is even.

- (a) (2 points) Prove the claim using a proof by contrapositive
- (b) (2 points) Prove the claim using a proof by contradiction

2. Equivalence (4 points)

Prove that the following are equivalent for all $x \in \mathbb{R}$:

- (i) x is rational, (ii) $x/2$ is rational, (iii) $3x - 1$ is rational.

3. Rational, irrational (6 points)

- (a) (2 points) Prove or disprove that if x and y are rational numbers, then x^y is also rational.
- (b) (2 points) Prove that $\sqrt[3]{2}$ is irrational.
- (c) (2 points) Prove that if x^3 is irrational, then x is irrational.

4. Sets (8 points)

- (a) (1 point) Find the sets A and B , if $A \setminus B = \{2, 4, 6, 8\}$, $B \setminus A = \{1, 5\}$ and $A \cap B = \{3, 7, 9\}$.
- (b) (1 point) Show that $A \times B \neq B \times A$ for all non-empty A and B , unless $A = B$.
- (c) (1 point) Can you conclude that $A = B$ if the sets A and B have the same power set?
- (d) (2 points) Prove $A \cap (A \cup B) = A$.
- (e) (2 points) Let $A_i = \{\dots, -2, -1, 0, 1, \dots, i\}$.
 - i. Find $\bigcup_{i=1}^n A_i$
 - ii. Find $\bigcap_{i=1}^n A_i$
- (f) (1 point) Find two sets A and B such that $A \in B$ and $A \subseteq B$.