CMPS/MATH 2170 Discrete Mathematics – Fall 13

9/3/13

2. Homework

Due **9/18/13** at the beginning of class

1. **Quantifiers (6 points)**
   Let $M(x) =$ "is a mathematician" and $P(x) =$ "knows how to program", and let the domain consists of all students at Tulane.
   (a) (3 points) Translate these statements into English.
      i. $\exists x : M(x)$
      ii. $\forall x : (M(x) \wedge P(x))$
      iii. $\forall x : (P(x) \rightarrow M(x))$
   (b) (3 points) Express each of the following statements in terms of $M(x), P(x)$, quantifiers, and logical connectives.
      i. It is not true that all students at Tulane know how to program.
      ii. No student at Tulane knows how to program.
      iii. Some student at Tulane does not know how to program.

2. **Negating Quantifiers (2 points)**
   Rewrite each of these statements such that all negation symbols immediately precede predicates P, Q, R, S.
   (a) $\neg[\exists x : (P(x) \wedge \neg Q(x))]$
   (b) $\neg[\exists x \exists y : (P(x, y)) \wedge \forall x \forall y : (Q(x, y))]$

3. **Negation (4 points)**
   For each of the propositions below, express it using symbolic notation, then negate this expression in symbolic notation, and finally translate the negated expression back into English.
   (a) Some drivers do not obey the speed limit.
   (b) Noone can keep a secret.

4. **Nested Quantifiers (6 points)**
   (a) (3 points) Let $F(x, y) =$"x can fool y", and let the domain be all people in the world. Use quantifiers to express each of the statements below.
      i. Everybody can fool Frank.
      ii. Everybody can fool somebody.
      iii. Everyone can be fooled by somebody.
   (b) (3 points) Express each of the statements below using predicates, quantifiers, logical connectives, and mathematical operators, where the domain consists of all integers.
      i. The product of two negative integers is positive.
      ii. There is a positive integer that is not the sum of three squares.

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5. **Formal Languages and Models (8 points)**

(a) (2 points each): For each of the following first-order sentences with the standard language for ordered number systems ($[+, *, 0, 1, <]$), state whether or not it is satisfied by the structures $\mathbb{R}, \mathbb{Q}, \mathbb{N}$, that is, the real numbers, the rational numbers, and the natural numbers (positive whole numbers, not including zero).

i. $\forall x \exists y : x < y$

ii. $\forall x \forall y : (x < y) \rightarrow (\exists z : x < z < y)$

(b) (2 points): Write a formula $\phi(x)$ that defines the even integers in $\mathbb{N}$, i.e., a formula for which $\phi(x)$ is true if and only if x is even.

(c) (2 points): Consider the following formula

$$\phi(x) := \forall y \forall z : (x = y * z) \rightarrow ((y = 1) \vee (z = 1)).$$

In $\mathbb{N}$, what subset does this formula define? I.e., for which $x \in \mathbb{N}$ is $\phi(x)$ true?