CMPS/MATH 2170 Discrete Mathematics – Fall 13

11/18/13

10. Homework

Due **12/2/13** at the beginning of class

1. **Dice game (6 points)**

   (a) (2 points) Consider playing the following game: You roll one loaded six-sided die, where the probability of rolling a six is 1/11 and the probability of rolling any other number is 2/11. When rolling a six you win \$20 when rolling any other number you lose \$3. Compute the expected win/loss of this game.

   (b) (2 points) Now consider playing the same game but rolling two loaded dice of the same type. For each six included in your result you win \$20 and for every other number you lose \$3. Use linearity of expectation to compute the expected win/loss of this game.

   (c) (2 points) Now consider playing the same game but rolling k loaded dice. Use linearity of expectation to compute the expected win/loss of this game.

   Clearly describe the sample space and the random variables you use. Half of the points will be given for correct notation. (The point of this exercise is to learn the notation, not just to get the intuition right.)

2. **Louisiana Lottery (6 points)**

   (a) (3 points) Consider the "Pick 3: Straight" game, which is played as follows: You pick an ordered tuple of three digits (out of $\{0, 1, \ldots, 9\}$; repetition is allowed). You pay \$1 to play. At the drawing, a random tuple of three digits is picked; repetition is allowed. If it matches your tuple of numbers exactly, you win \$500 (so, that's a net win of \$499 since you paid \$1 to play).

      i. What is the expected win/loss of this game?

      ii. Consider the success probability of winning. Apply the formula for the expected value of the geometric distribution to calculate the expected number of times you need to play this game until you win for the first time.

   (b) (3 points) Now consider the "Pick 3: Box" game, which is played as follows: You pick an unordered set of three unique digits (out of $\{0, 1, \ldots, 9\}$; this time, repetition is not allowed). You pay \$1 to play. At the drawing, a random tuple of three digits is picked; repetition is allowed. If the tuple of numbers contains exactly your three numbers, you win \$80 (so, that's a net win of \$79 since you paid \$1 to play).

      i. What is the expected win/loss of this game?

      ii. Consider the success probability of winning. Then apply the formula for the expected value of the geometric distribution to calculate the expected number of times you need to play this game until you win for the first time.

   Which of the two "Pick 3" games would you rather play?

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(c) (2 bonus points) Now, let's play lotto: You pick a set of 6 unique numbers out of 40 numbers; without repetition. You pay \$1 to play. At the drawing, a random set of 6 unique numbers is drawn out of 40 numbers; without repetition. If your set of numbers matches the drawn numbers, you win \$450,000. If your set of numbers matches five of the drawn numbers, you win \$2,000. If your set of numbers matches four of the drawn numbers, you win \$50. If your set of numbers matches three of the drawn numbers, you win \$3. What is the expected win/loss of this game?
(This follows a hypergeometric distribution. Think of constructing winning sets of numbers by drawing from six winning numbers and then drawing from 34 losing numbers.)

3. **Conditional Probability (3 points)**
Consider the experiment that two fair dice are rolled. Let E be the event that the first die is a 6. Let F be the event that the sum of the dice is 9.

   What are $P(E)$, $P(F)$, $P(E|F)$? Are E and F independent?

4. **Independence (3 points)**
Two random variables X and Y on a sample space S are *independent* if

$$P(\{X = a\} \cap \{Y = b\}) = P(\{X = a\}) \cdot P(\{X = b\}) ,$$

   for all $a, b \in \mathbb{R}$, where the event $\{X = a\} = \{s \in S \mid X(s) = a\}$.

   Let X and Y be the random variables that count the number of heads and the number of tails that come up when two coins are flipped. Show that X and Y are not independent.

5. **Variance (3 points)**
Let X be the category of a hurricane that has hit the US mainland. Hurricanes of a higher category have higher wind speeds. Assume the probability distribution for X is as follows:
$P(X = 1) = 0.372$,
$P(X = 2) = 0.229$,
$P(X = 3) = 0.288$,
$P(X = 4) = 0.098$,
$P(X = 5) = 0.013$.
What are the expected value, variance, and standard deviation of X?