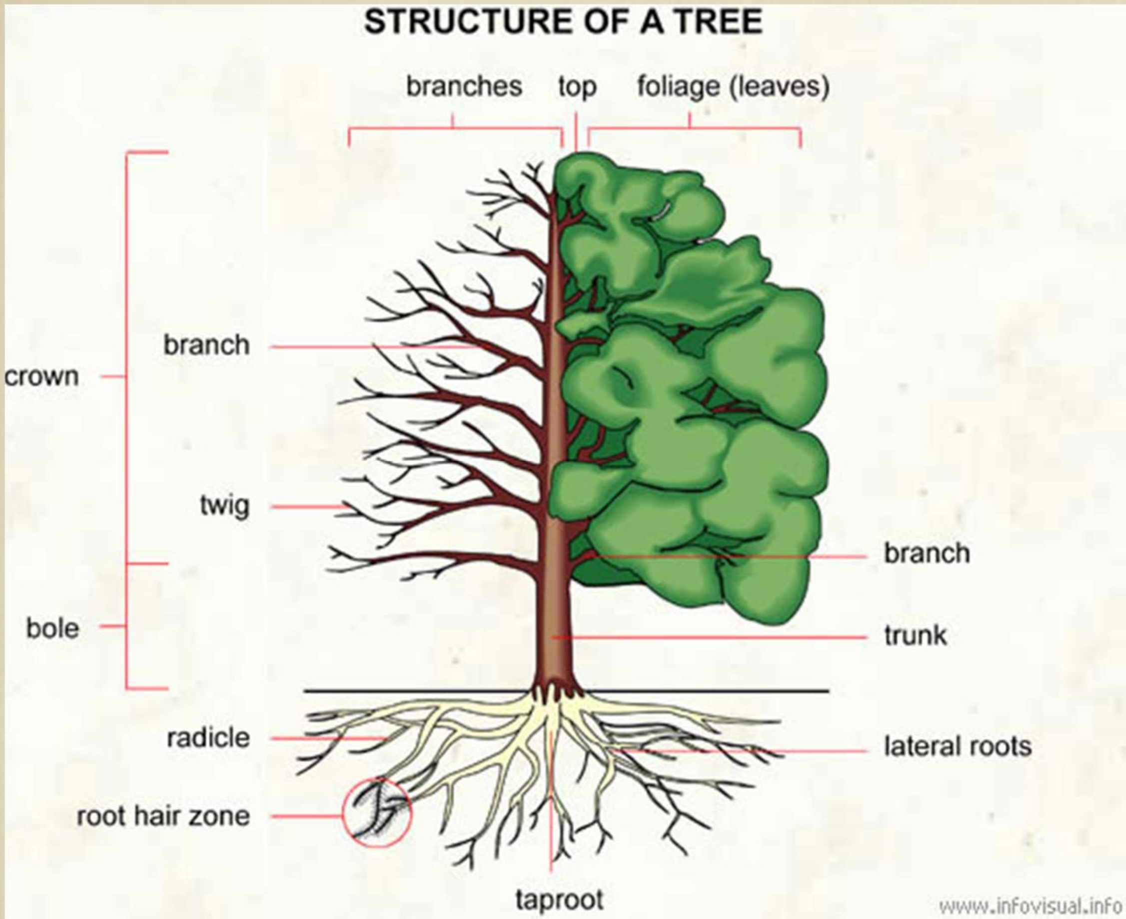
Linked Structures Songs, Games, Movies Part III

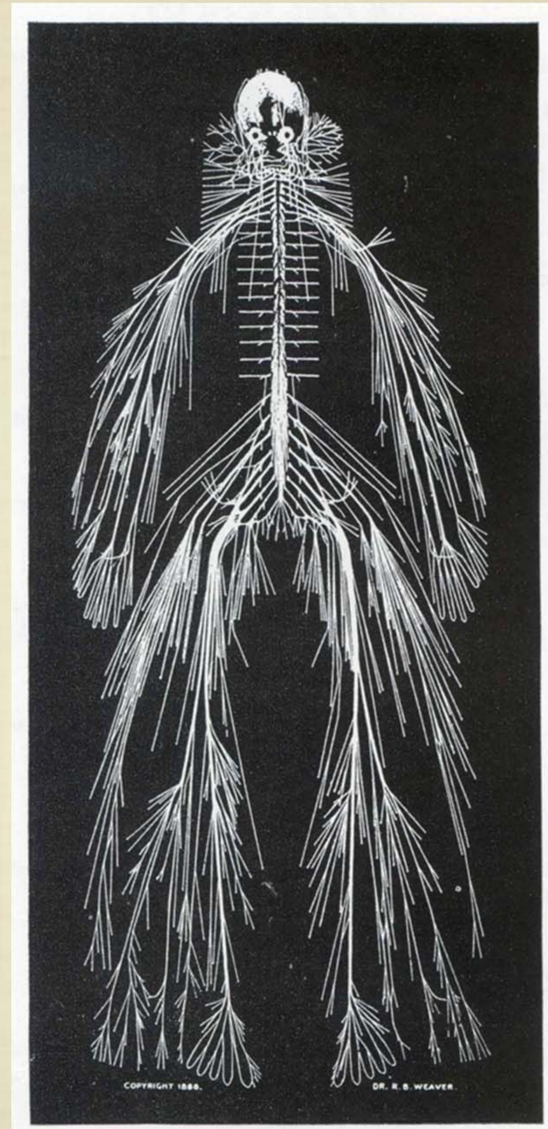
Fall 2013

Carola Wenk

Biological Structures

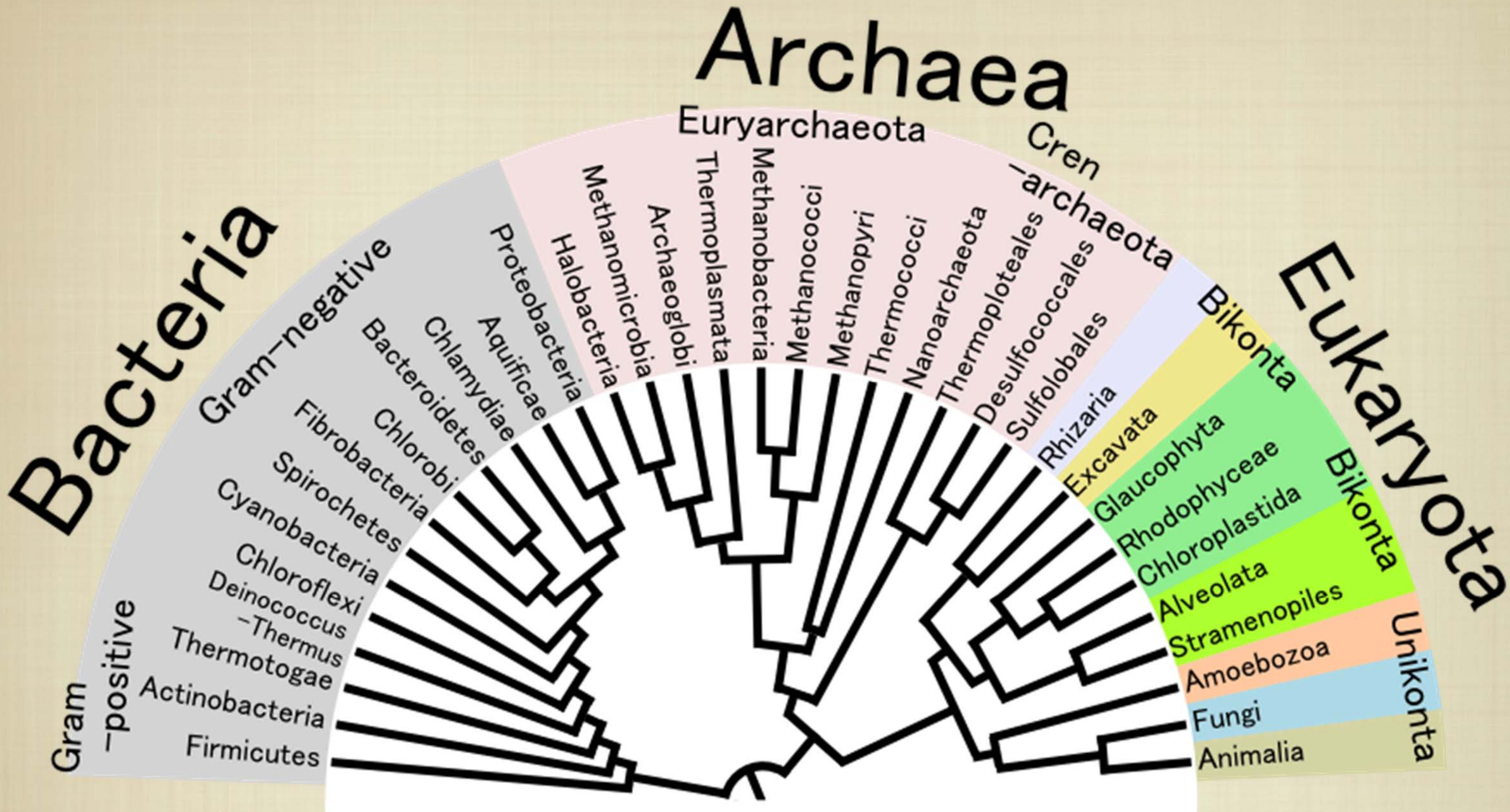


Nature has evolved vascular and nervous systems in a hierarchical manner so that nutrients and signals can quickly reach their destinations.



Human Nervous System

A large amount of information can be viewed more easily when it is laid out in a hiearchical fashion.



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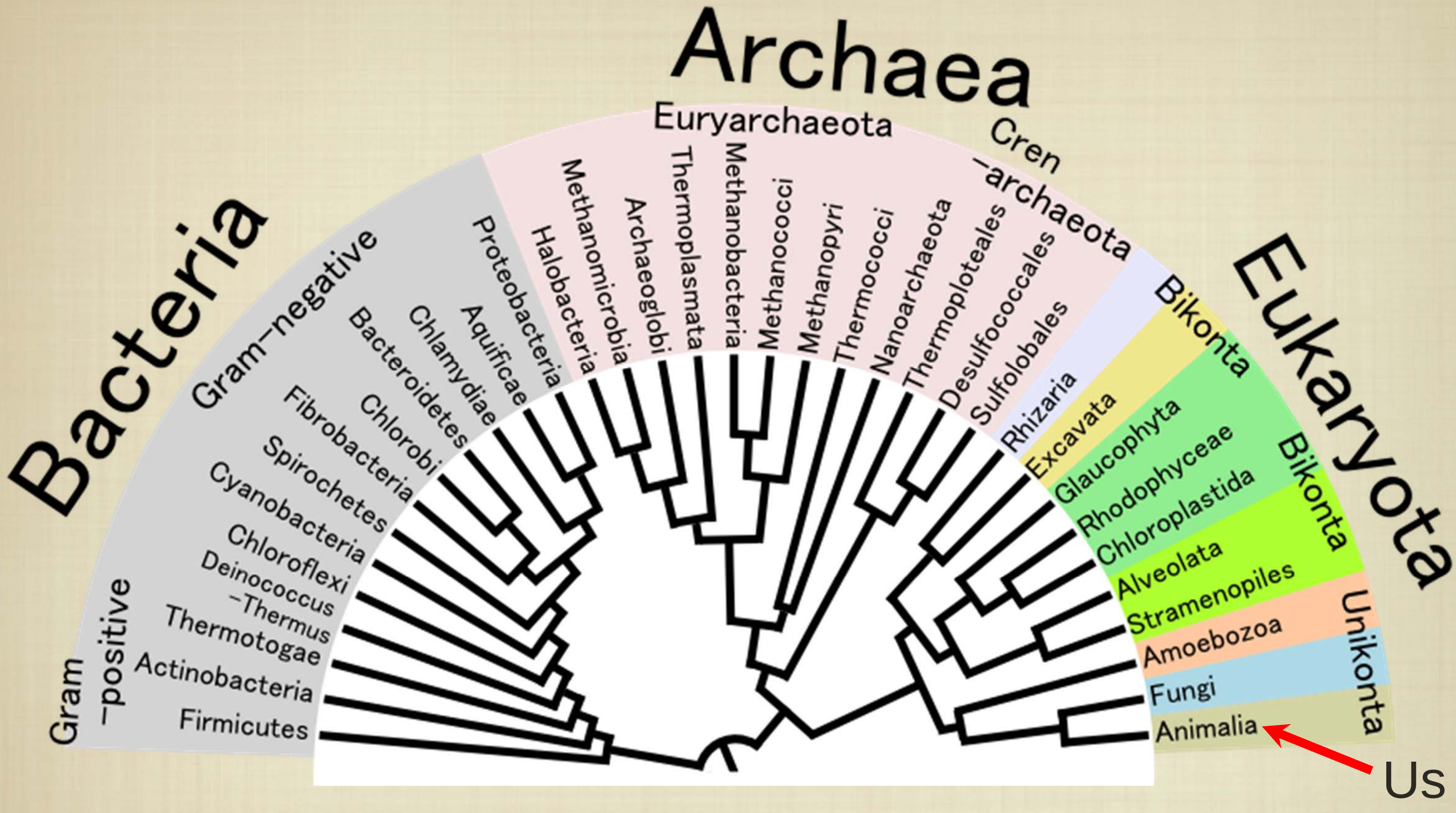

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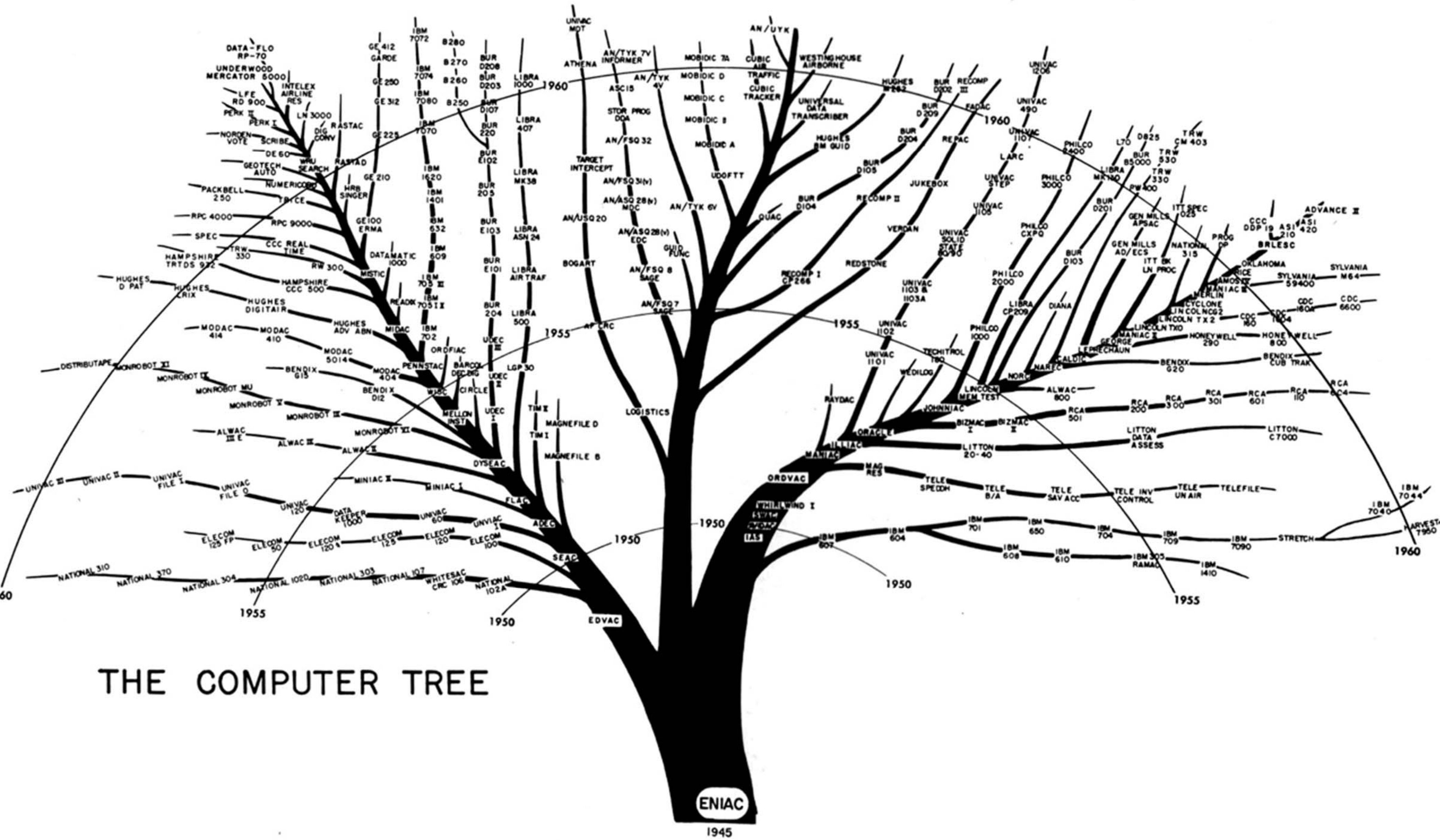

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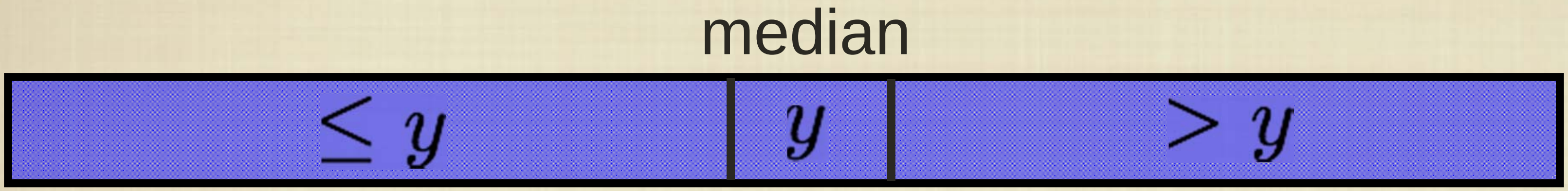
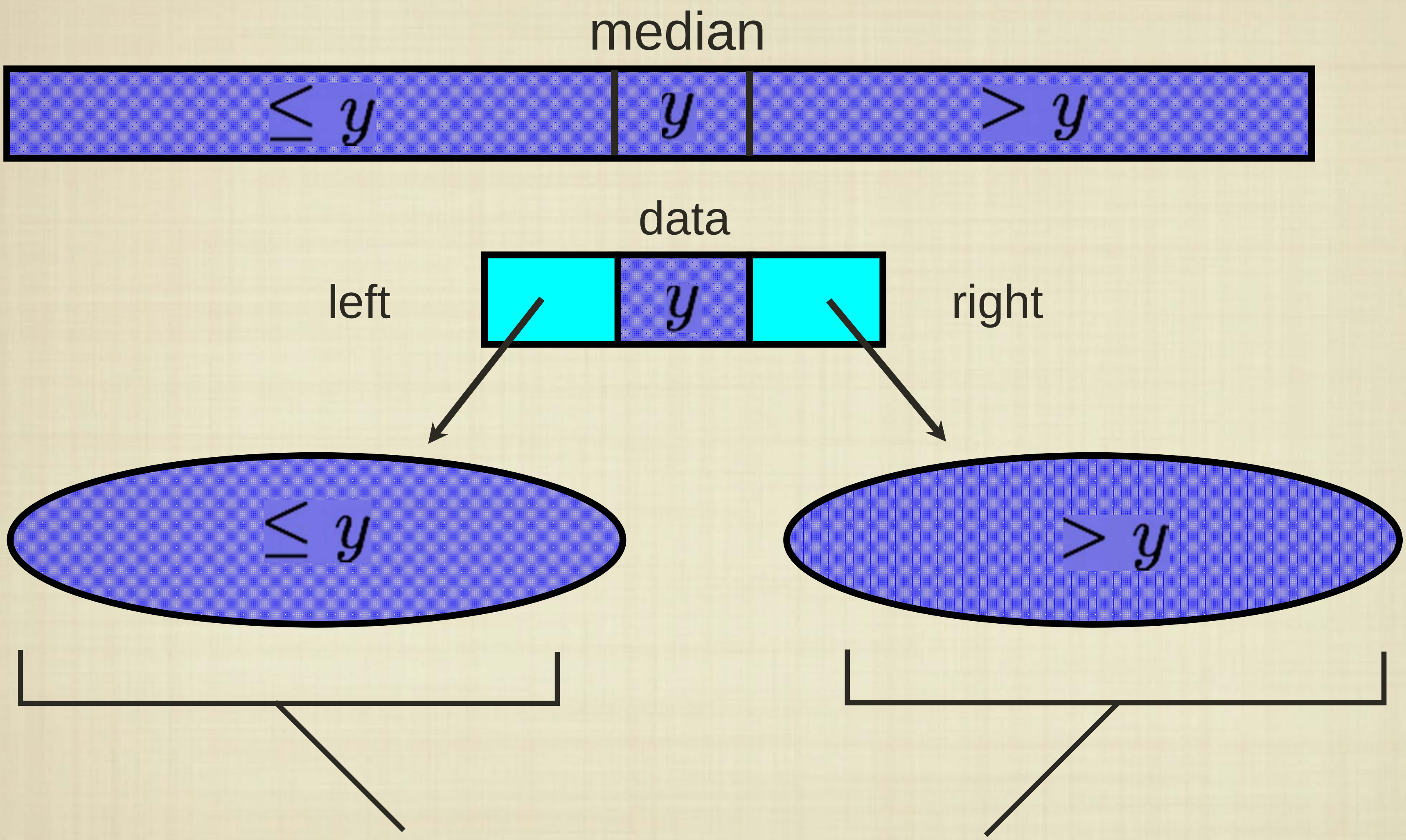
Remember Binary Search?



In binary search, immediate access to the median allowed us to quickly direct our search to smaller or larger elements.

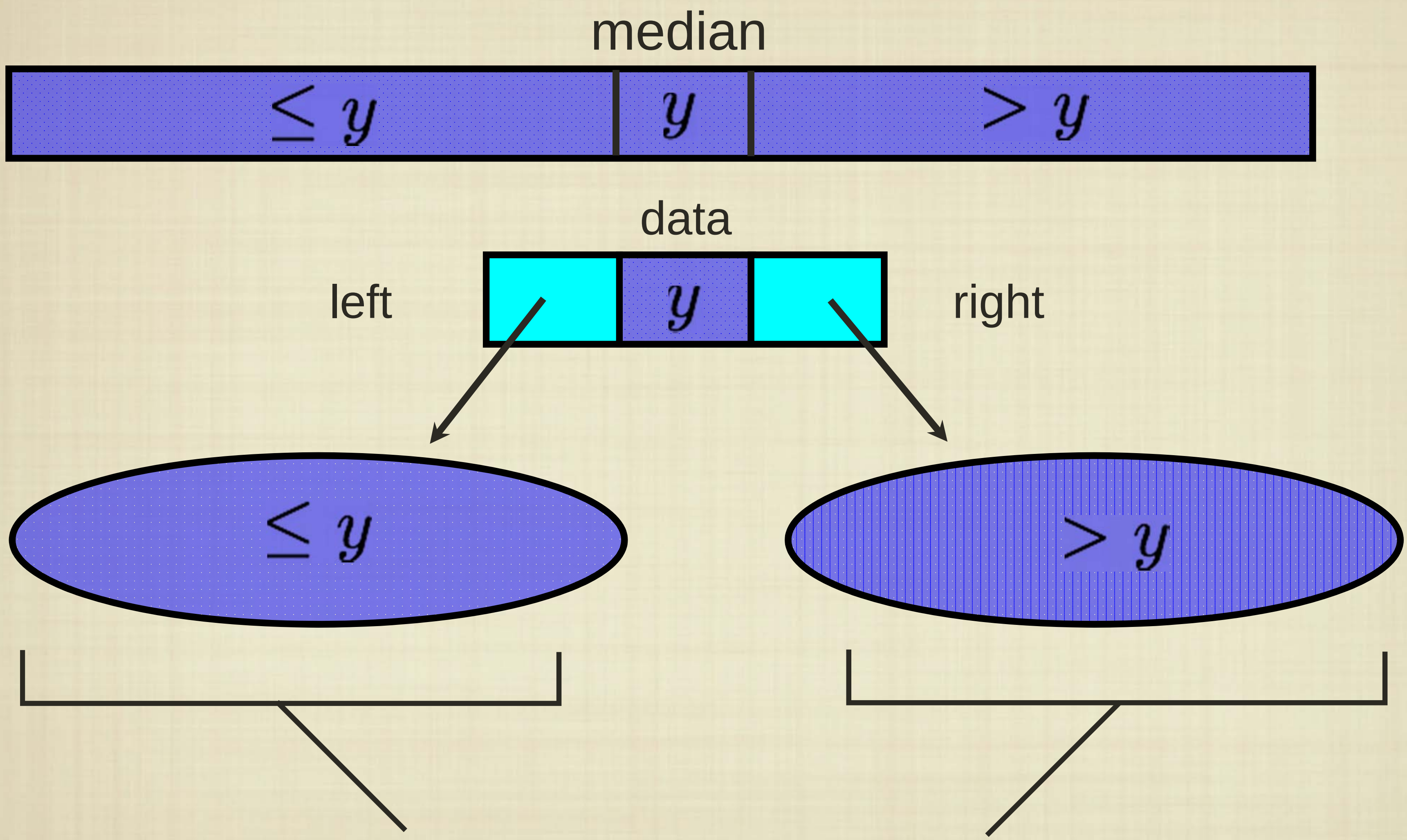
Can we use this idea to develop a linked structure?

Remember Binary Search?



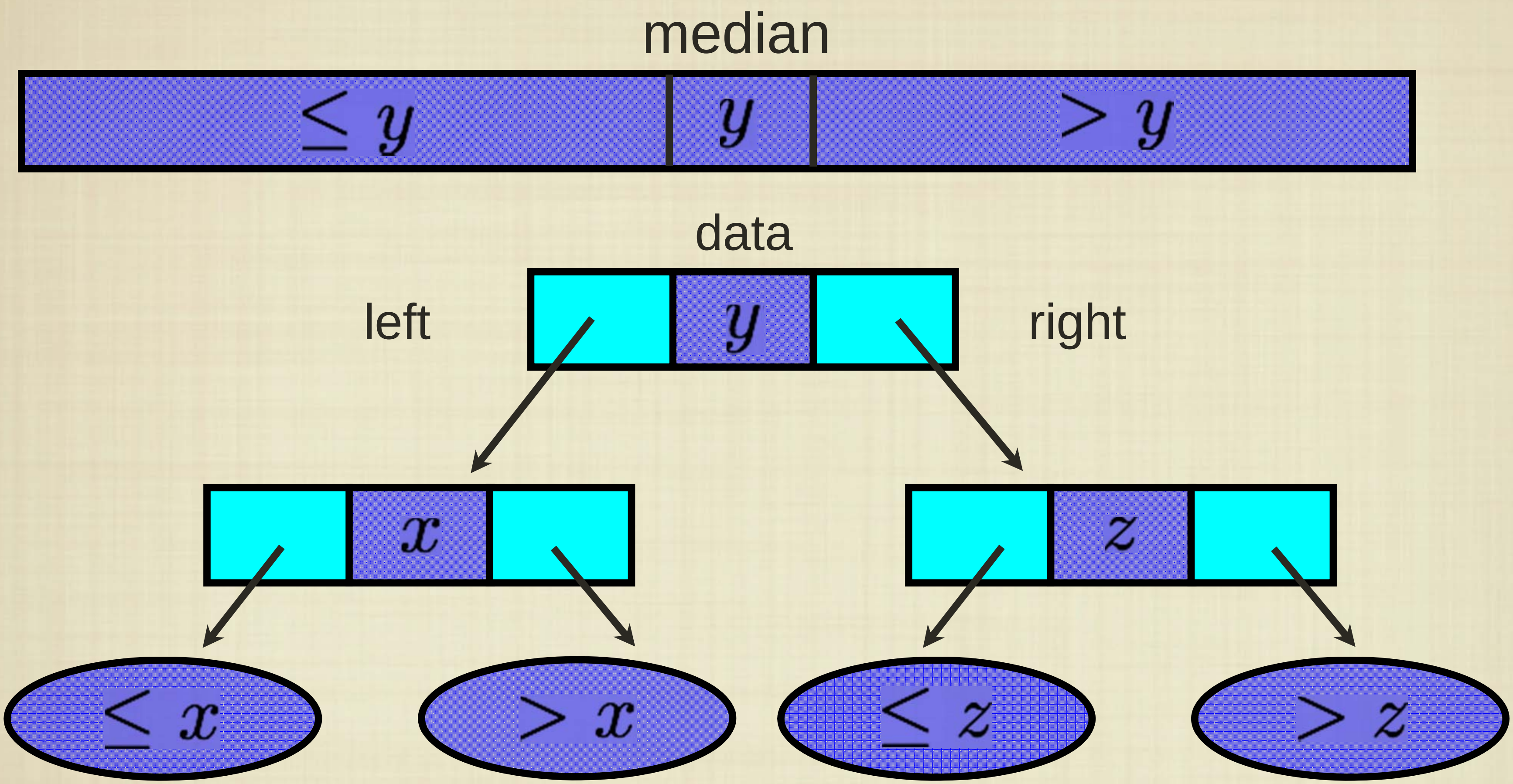
- We can split the list into two “halves” of a linked structure.

Remember Binary Search?



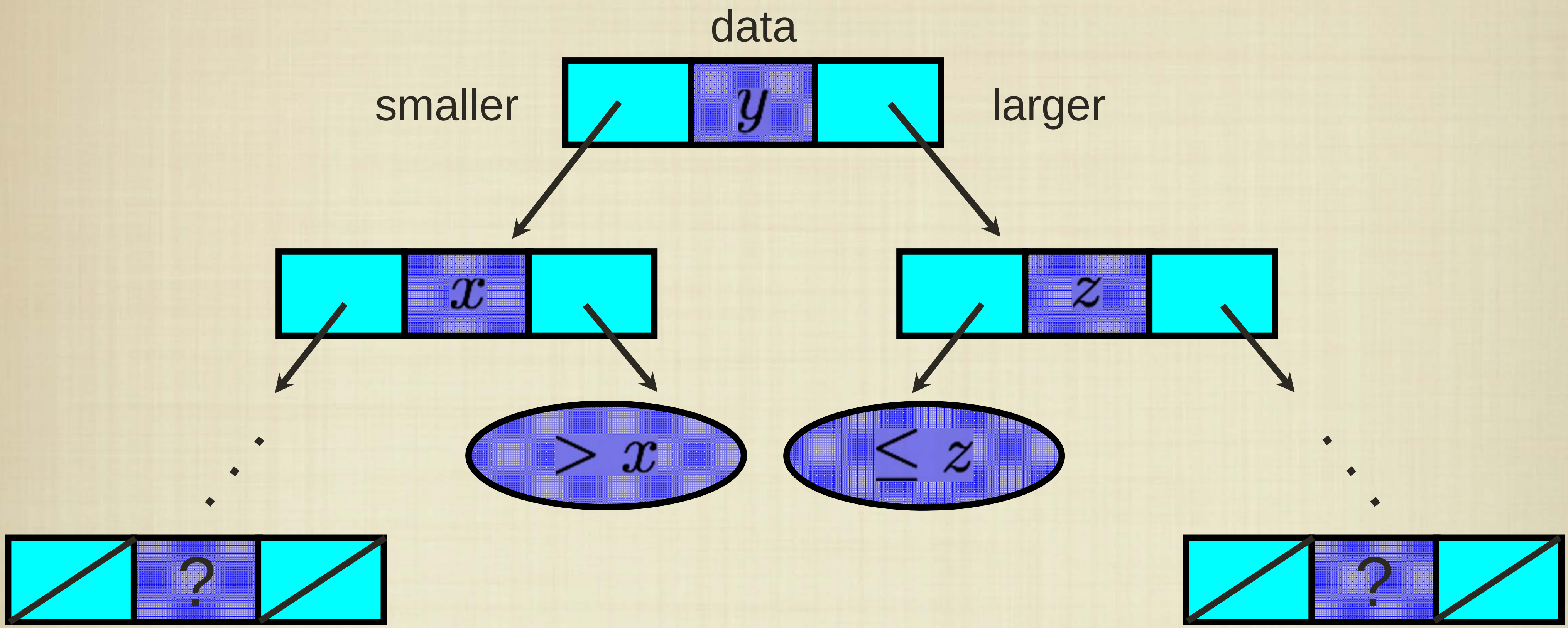
The two halves of a binary search tree can be defined recursively.

Binary Search Trees



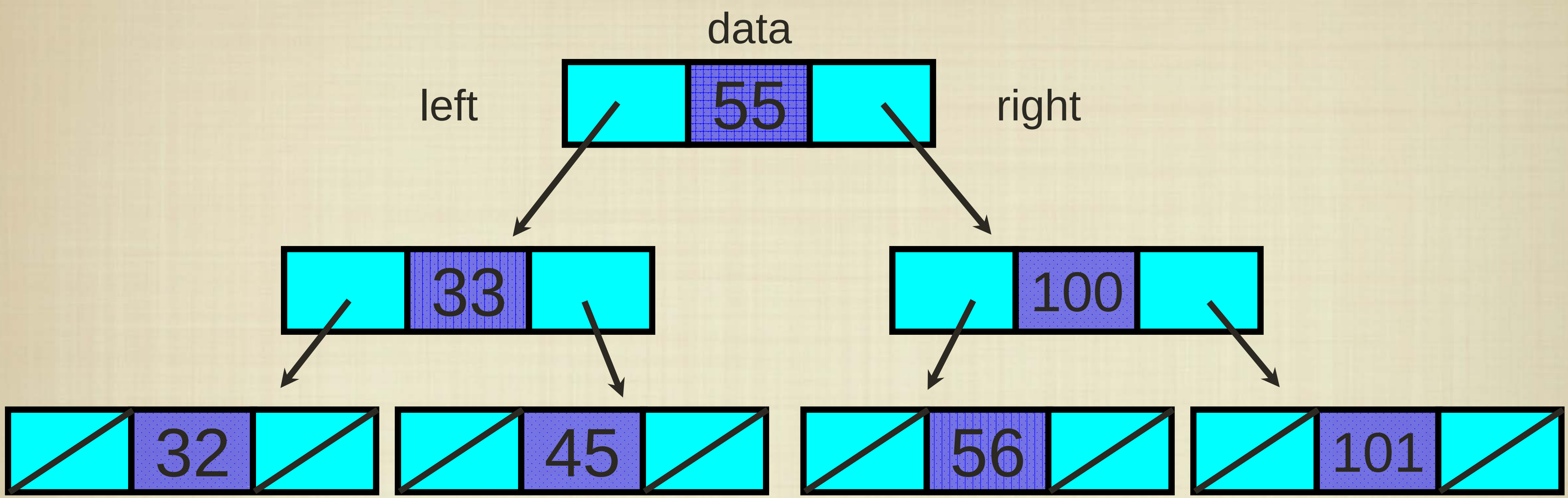
- How do we define this type of structure in Python?

Binary Search Trees



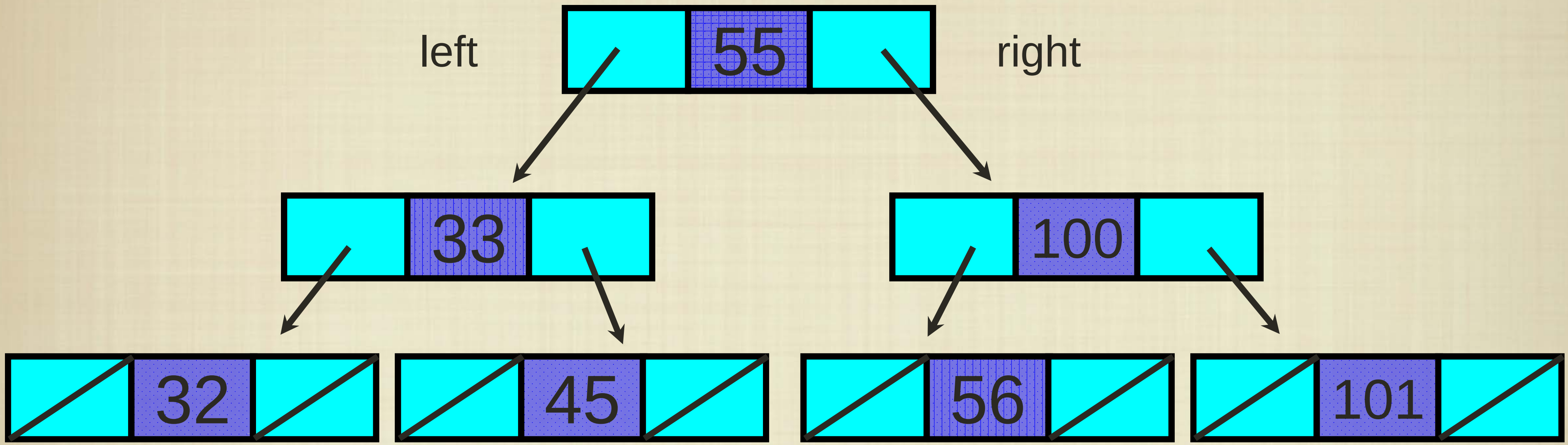
- A node in this linked structure is called a “leaf” if it has no “children.” The topmost element is the “root”.

Binary Search Tree Example



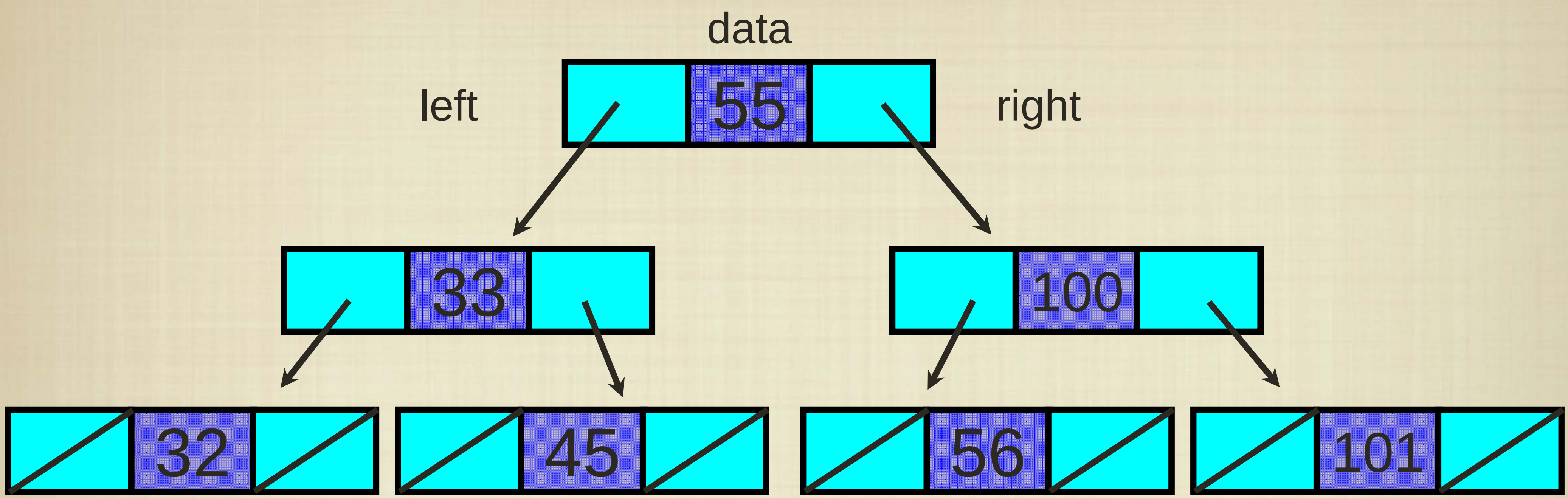
- Where are the minimum and maximum of the element?
- Can we quickly find any element?
- What about adding/removing?

Finding the Minimum/Maximum



- The minimum element can be found by following the `left` references, and the maximum element can be found by following `right` references.

Finding Any Item (Quickly)



- In exactly the same way as binary search, we can recursively focus on one “side” of the tree by checking the root element. How long does this take?

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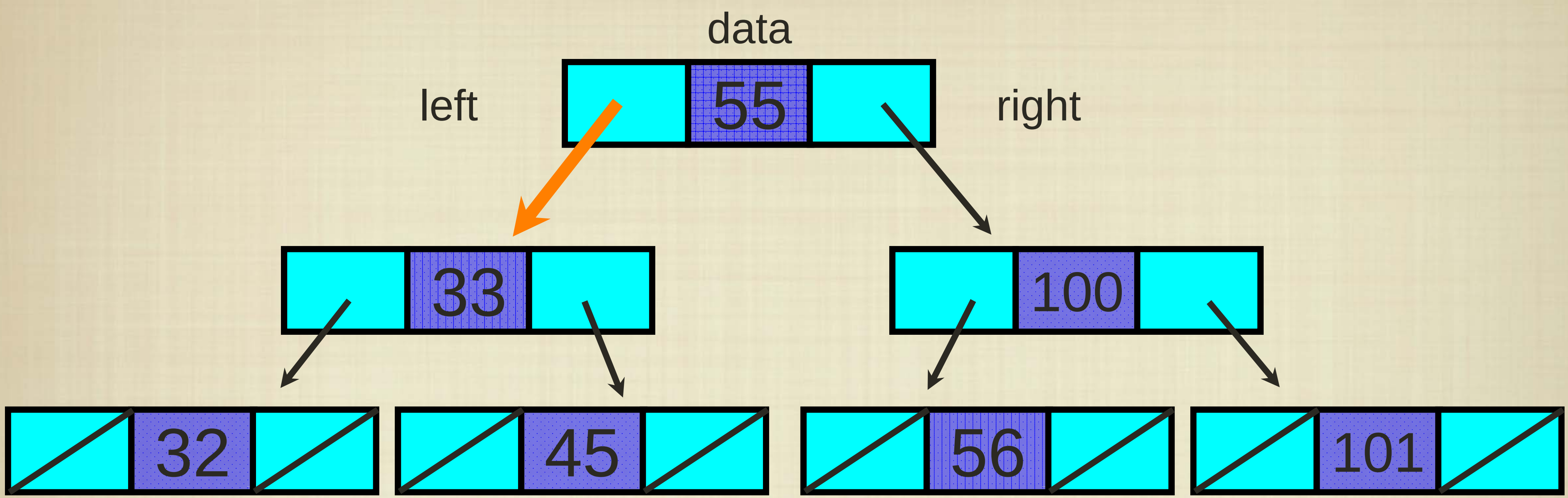

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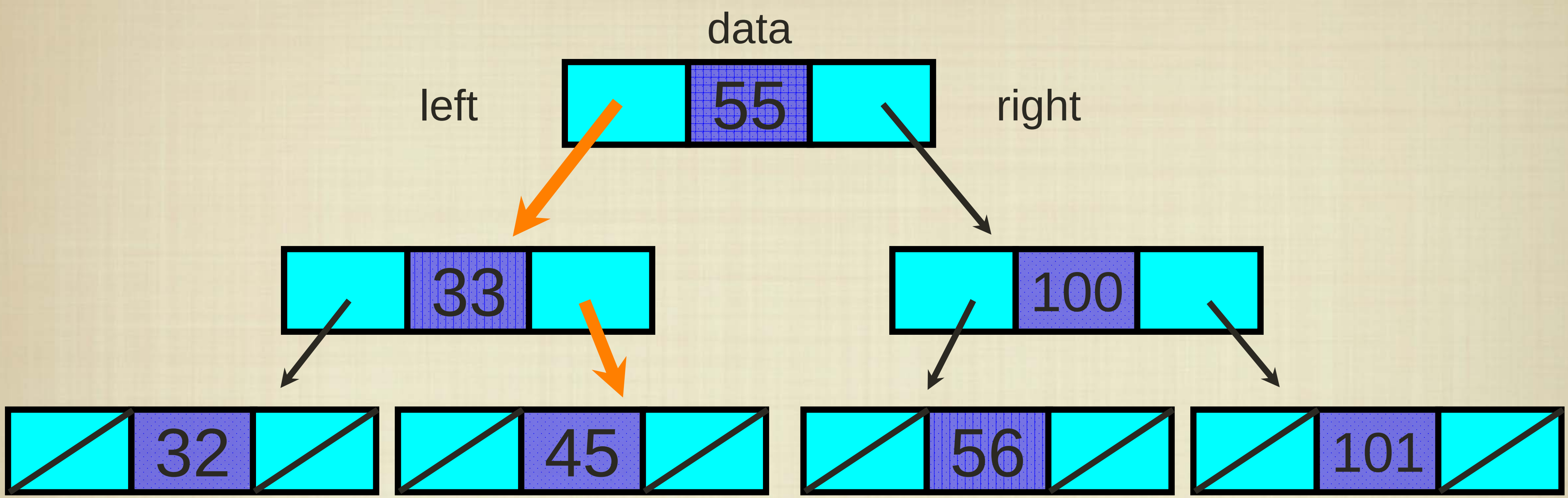
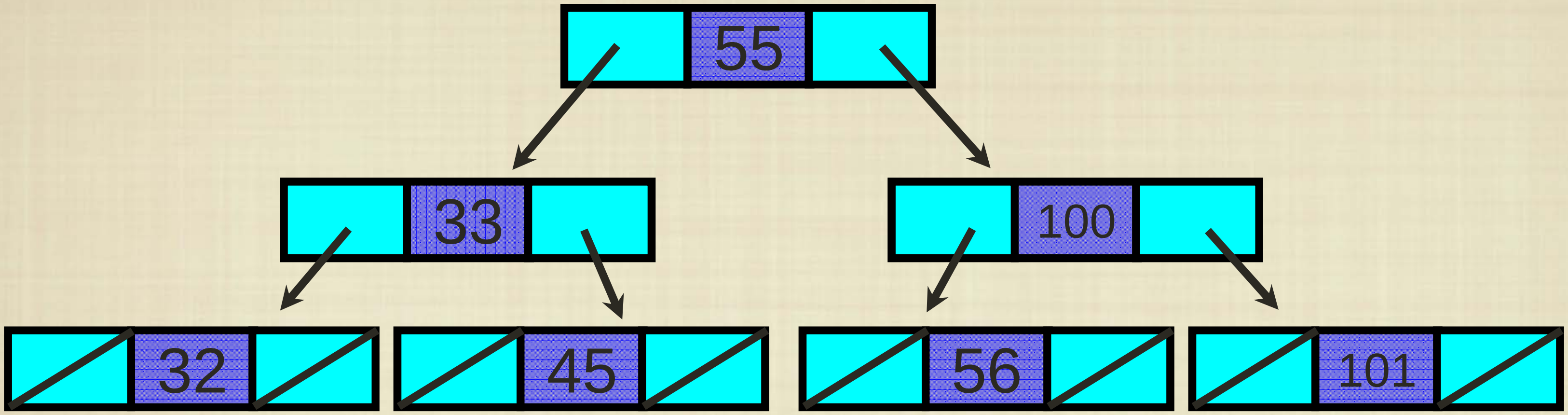


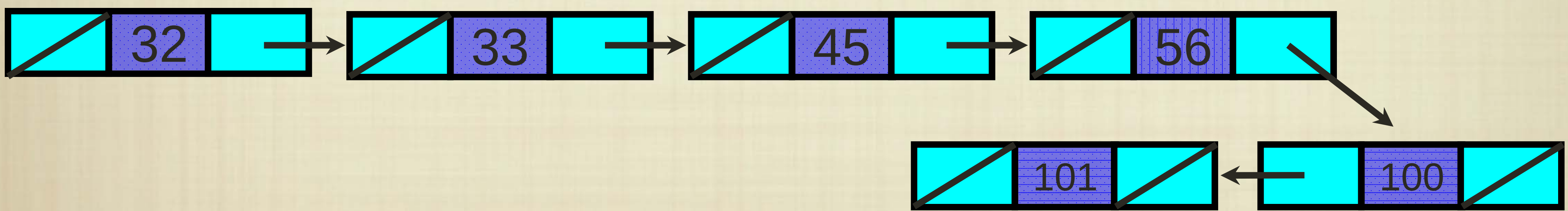
- In exactly the same way as binary search, we can recursively focus on one “side” of the tree by checking the root element. How long does this take?

The (worst-case) time to find an item depends on the height of the tree. How large can the height be?

Worst Case Scenario

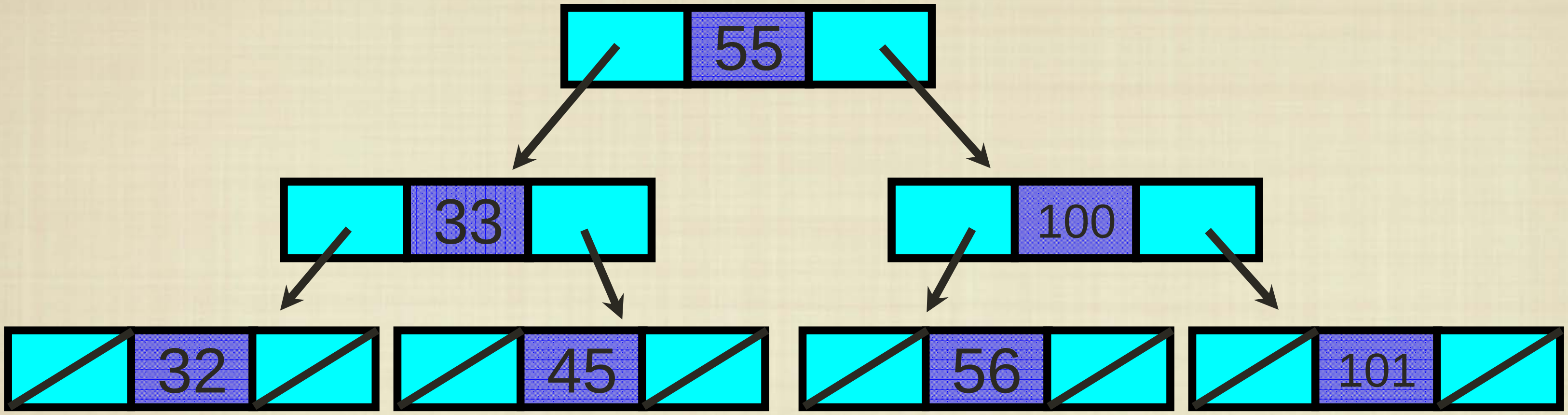


- In this particular tree, we can find any element in two steps.



Is this a tree? Why or why not?

Worst Case Scenario



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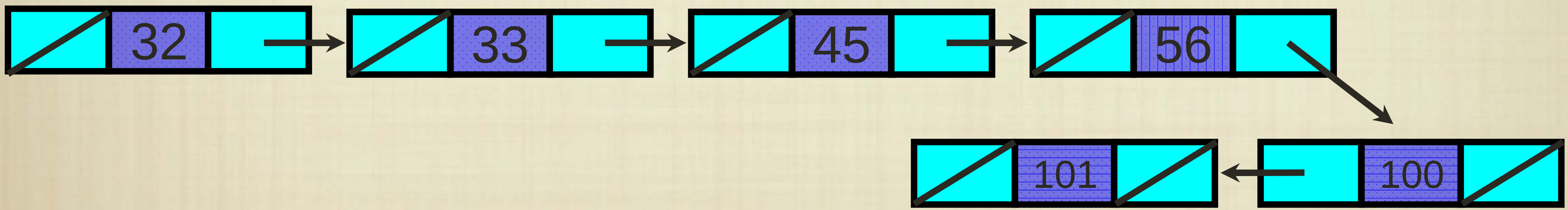
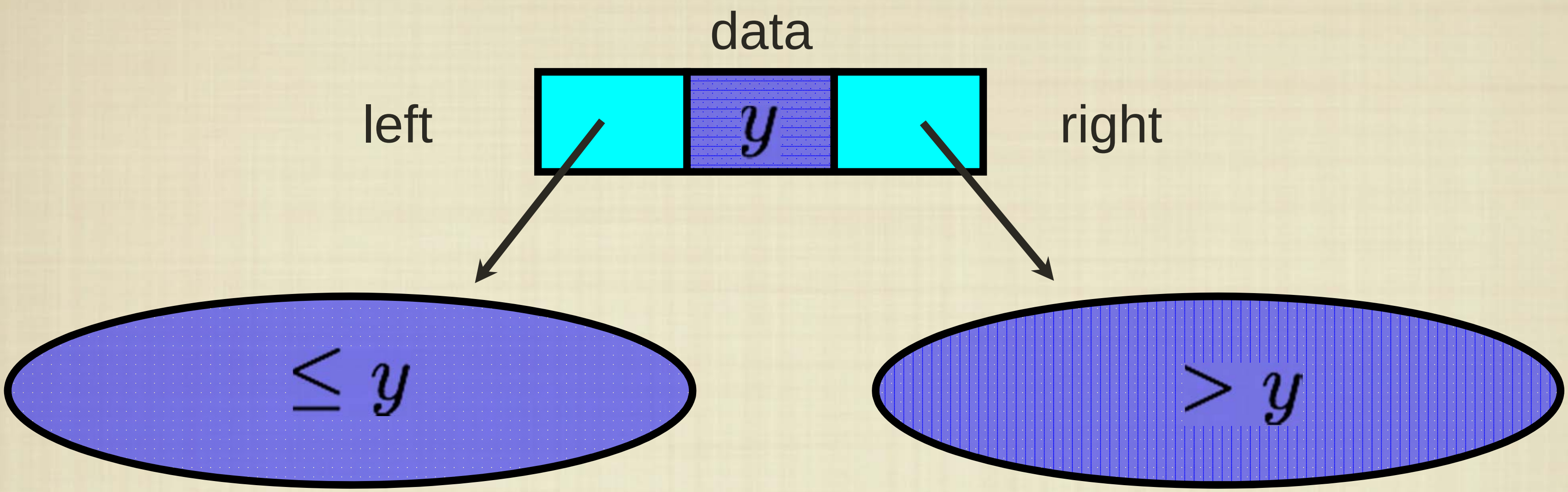


This is a binary tree that's not very tree-like. Why would a tree look like this?

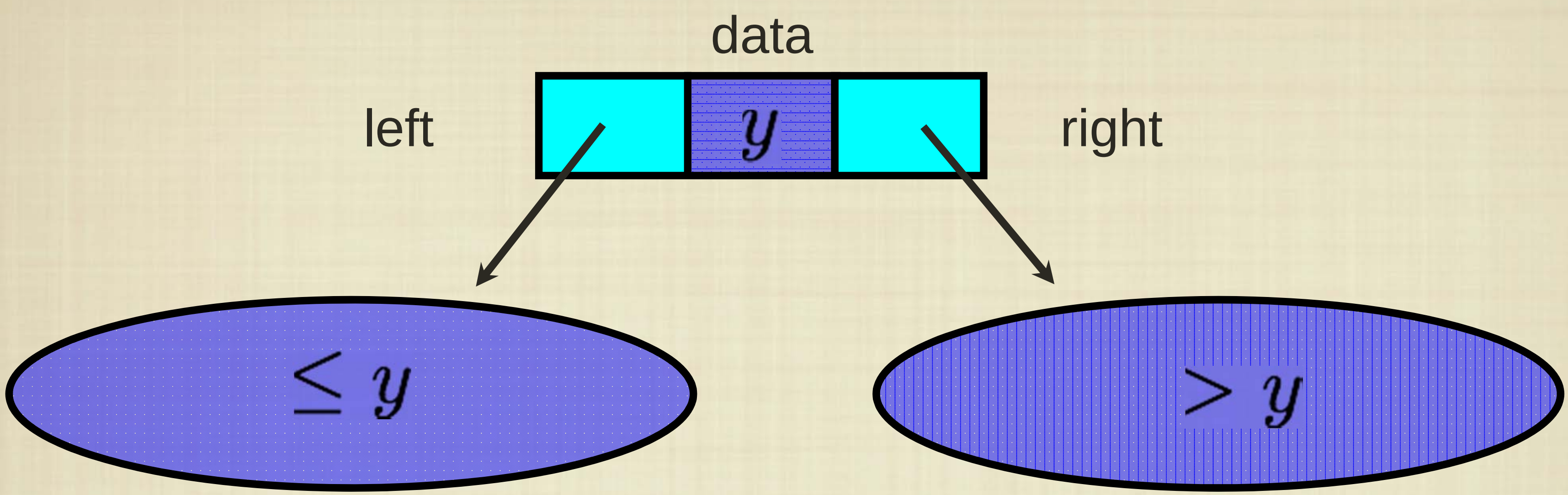
Adding Items



According to our definition, we know at least which side of the tree to insert.

Algorithm: Recursively determine which side of the tree to insert, and create a new element at the bottom.

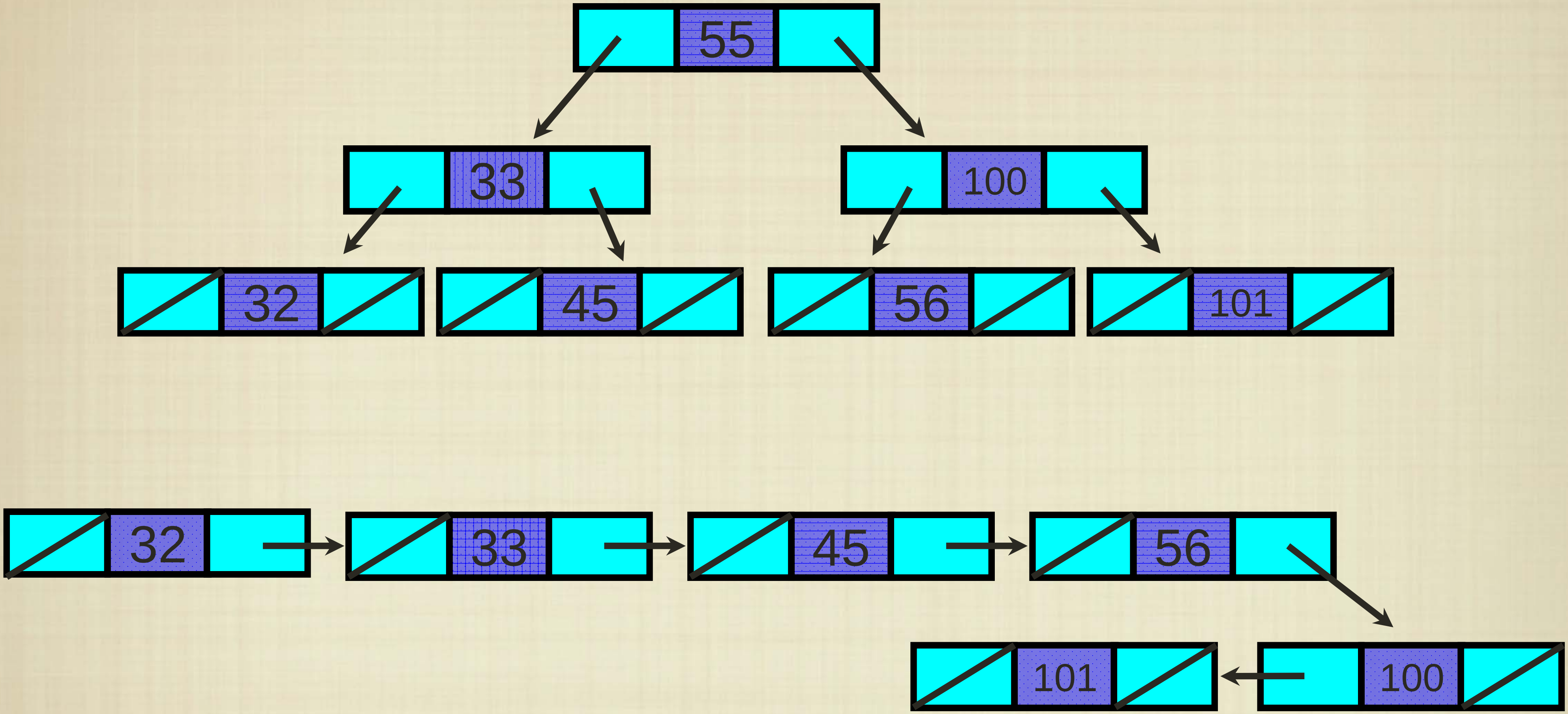
Adding Items



Algorithm: Recursively determine which side of the tree to insert, and create a new element at the bottom.

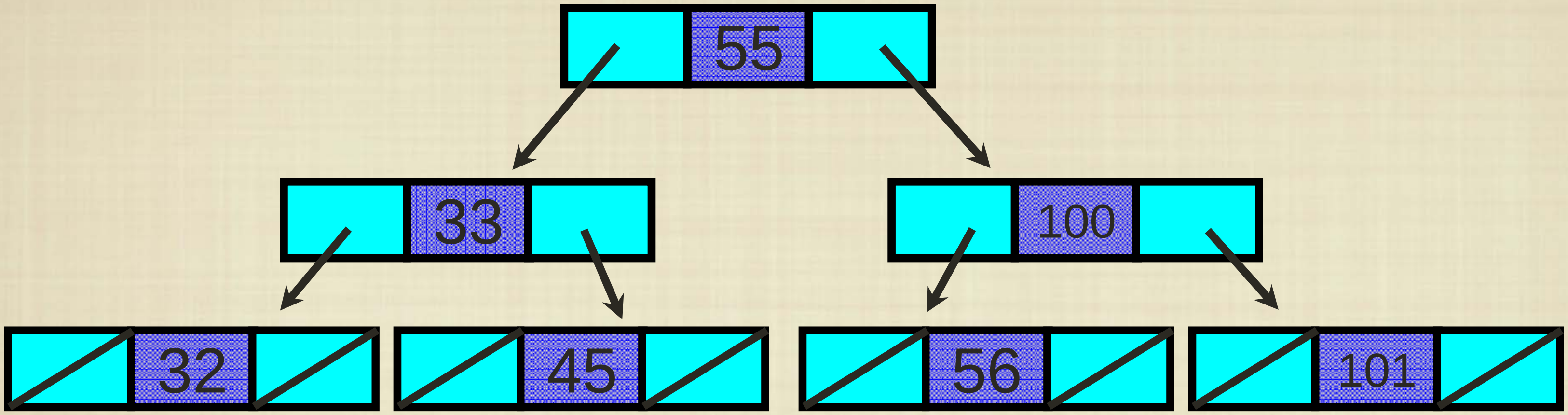
Unfortunately, we can't control the order in which things are added.

Worst Case Scenario

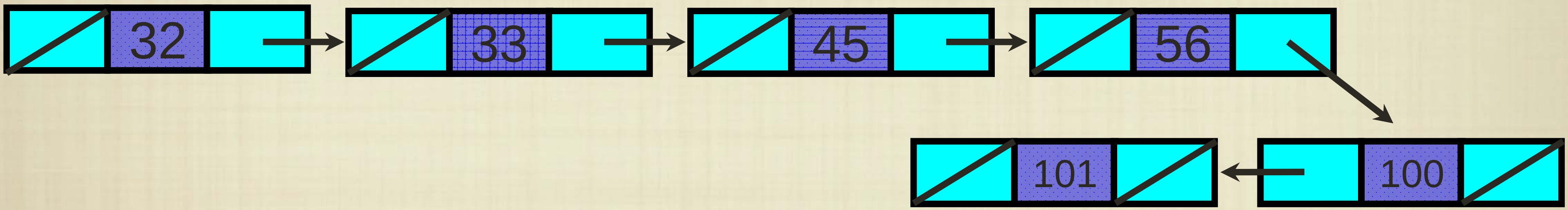


How were these two trees created?

Worst Case Scenario



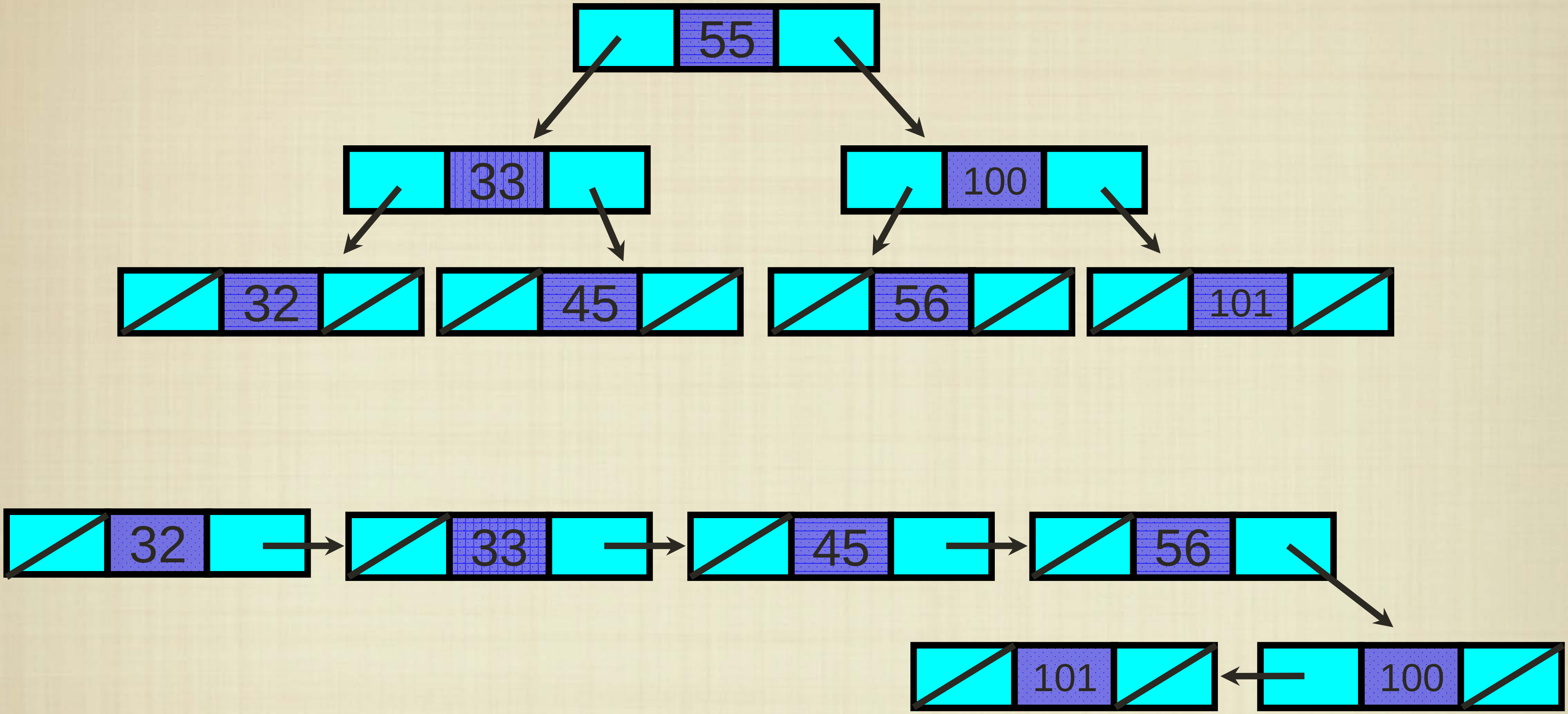
Insertion Order: 55, 33, 100, 32, 45, 56, 101



Insertion Order: 32, 33, 45, 56, 100, 101

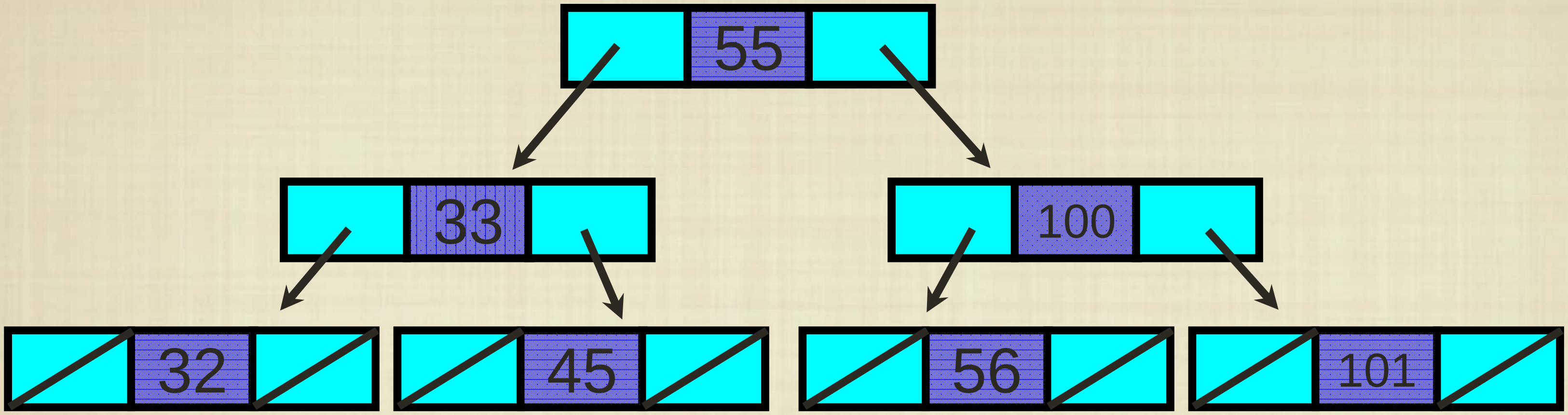
Ironically, inserting items in sorted order produces an extremely “imbalanced” tree.

Worst Case Scenario



The <u>height</u> of a binary search tree is the longest root-to-leaf path; researchers have studied how to minimize this.

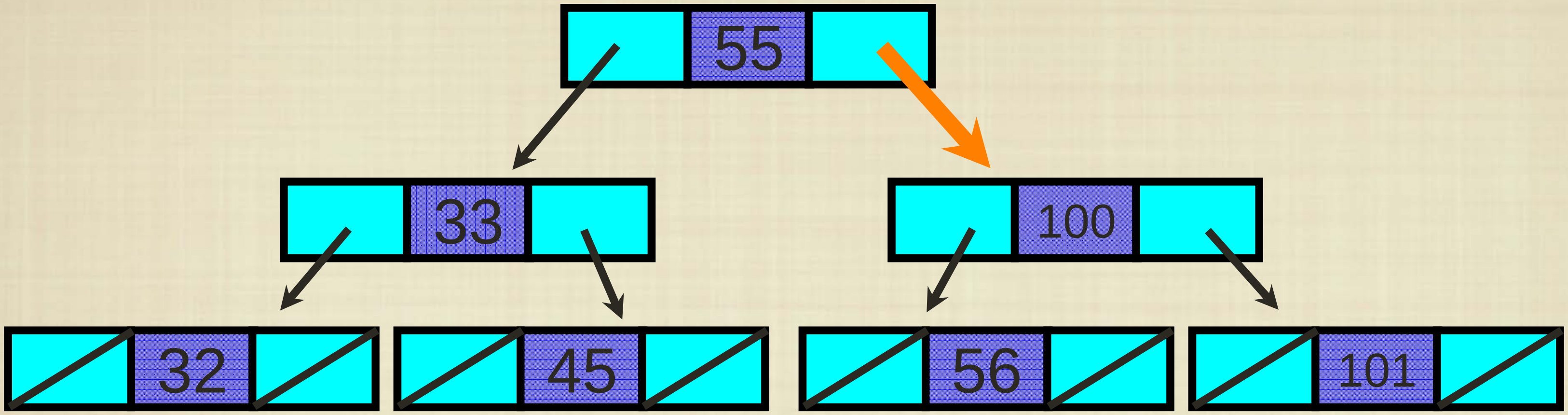
Removing Items



We may want to remove any item in the tree - what if we want to delete the “root”?

We need to find a substitute; where is the next largest item in the tree located?

Removing Items



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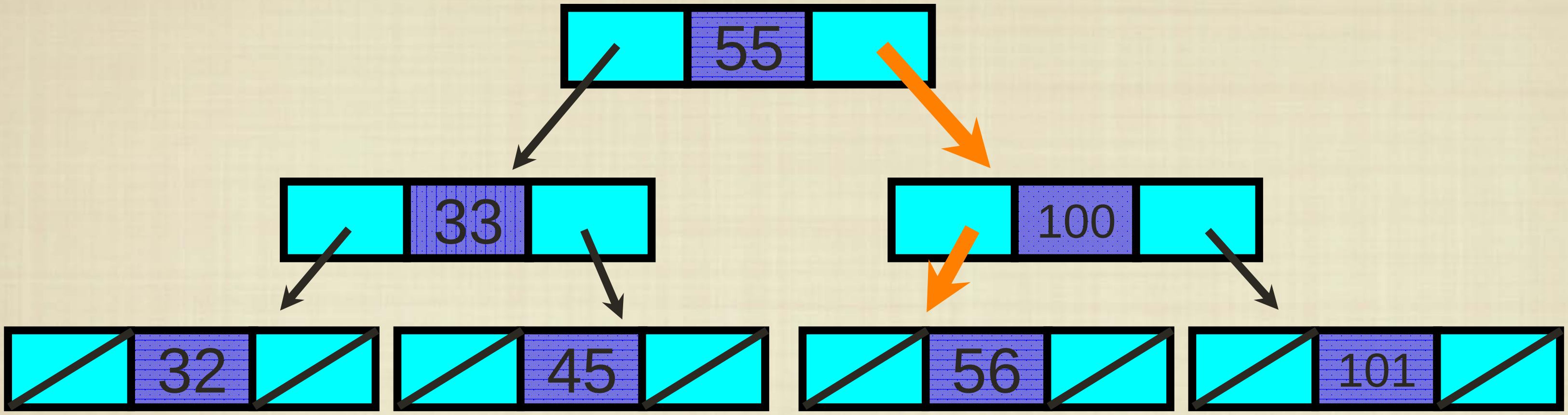
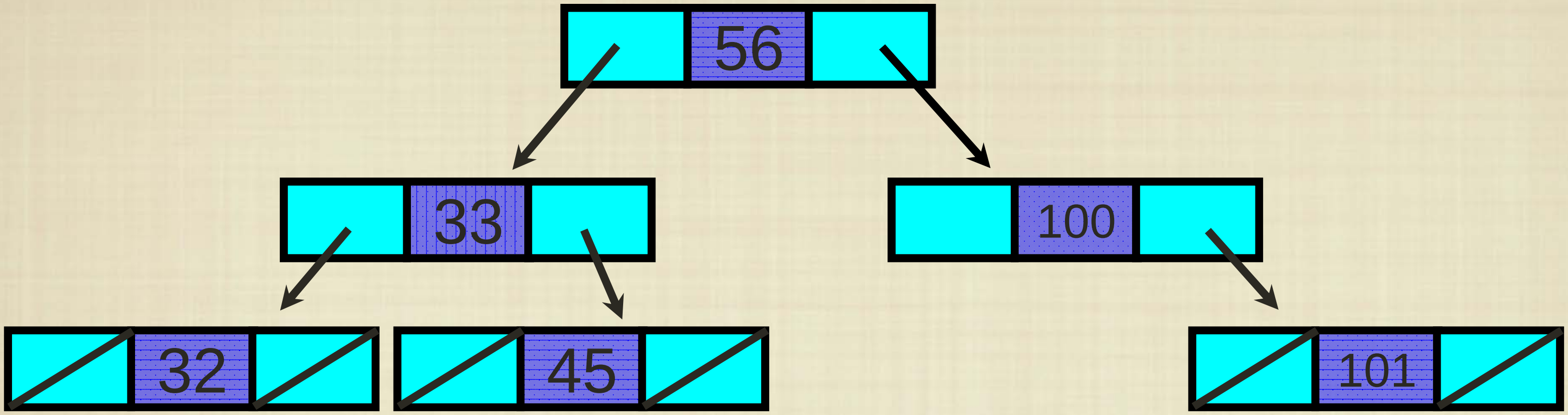


Which two elements can take the place of the item to be removed?

Algorithm: Replace the item to be deleted with the smallest item larger than it. (The minimum element in the right subtree.)

Removing Items



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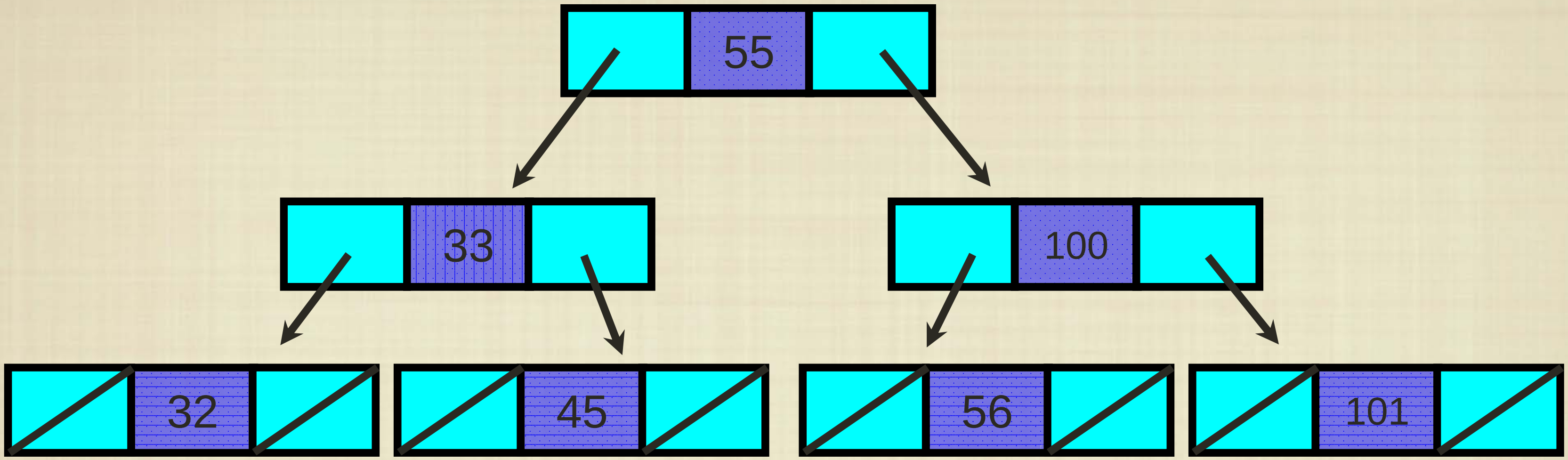
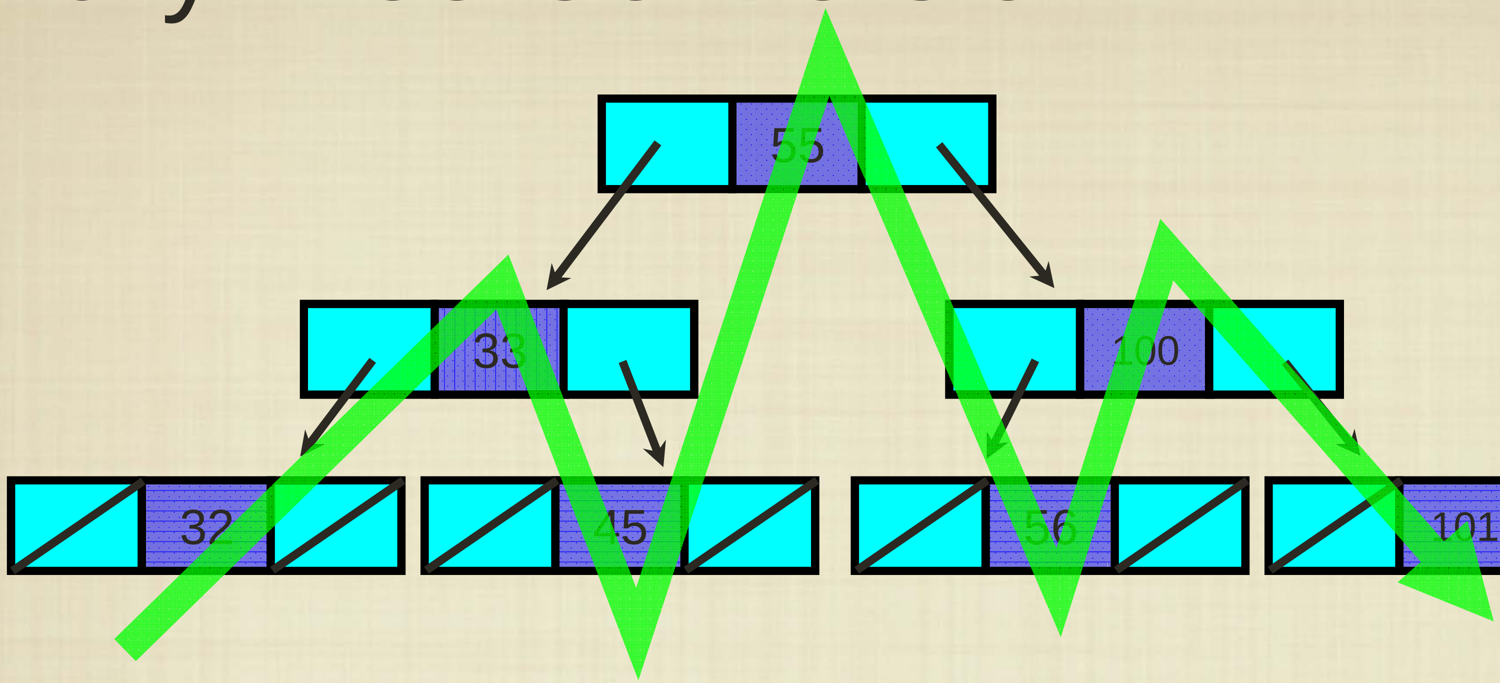
Binary Tree Conversion



- Suppose I gave you a tree, how would you convert it into a sorted array?
- What if I wanted to save the tree to a file and reconstruct it exactly?

Binary Tree Conversion



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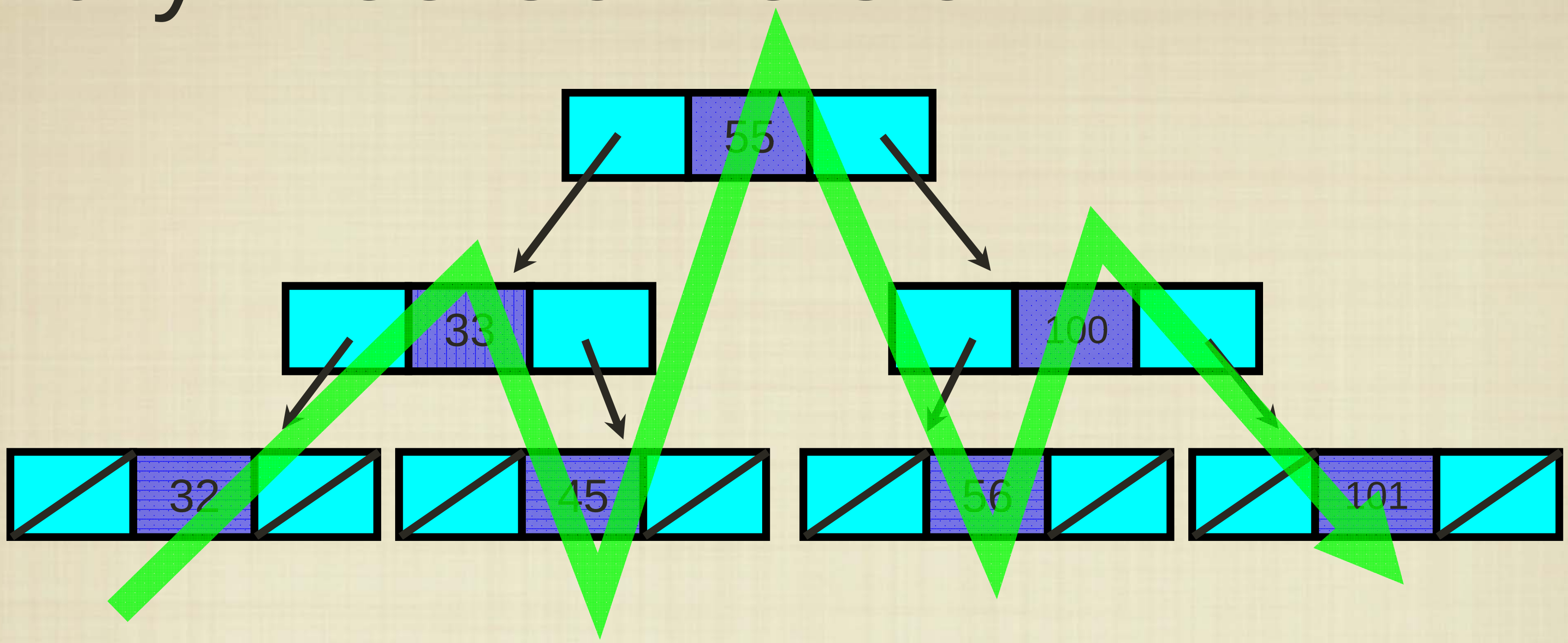

- An <u>in-order traversal</u> works by “visiting” the left subtree, then the current item, and then visiting the right subtree.

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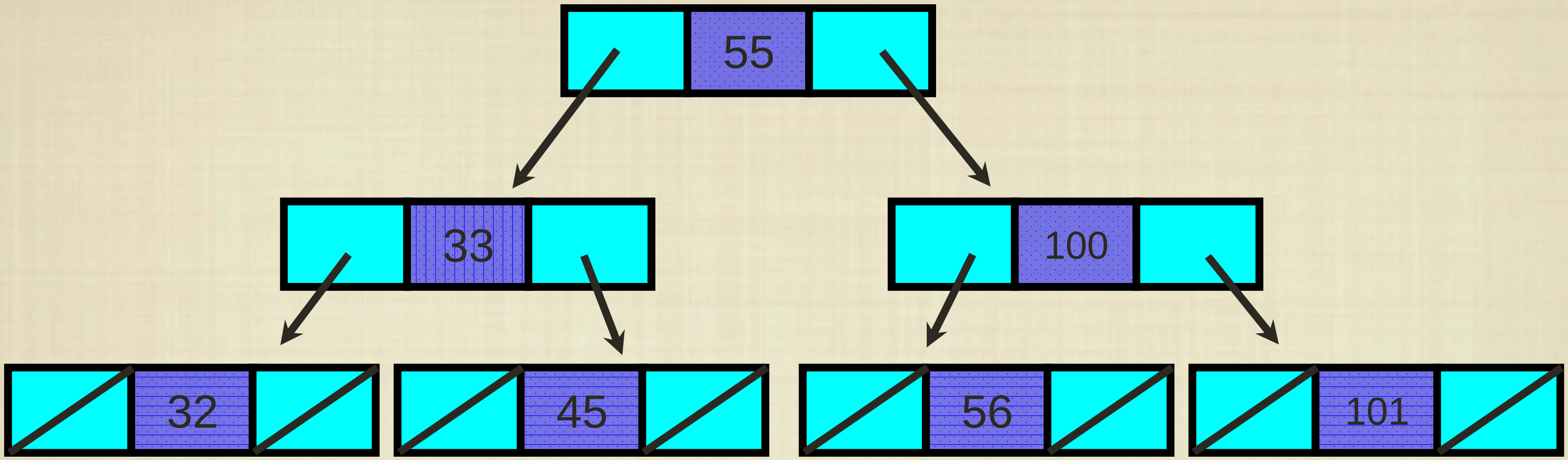

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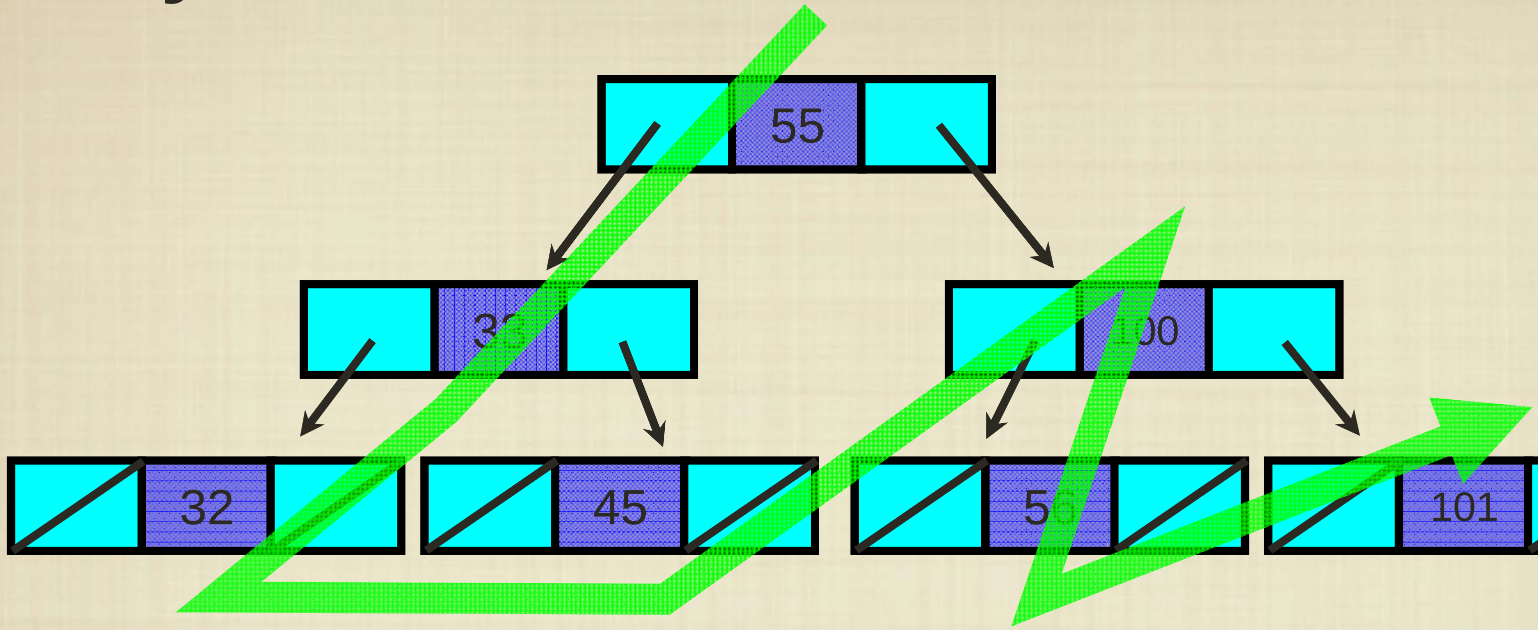
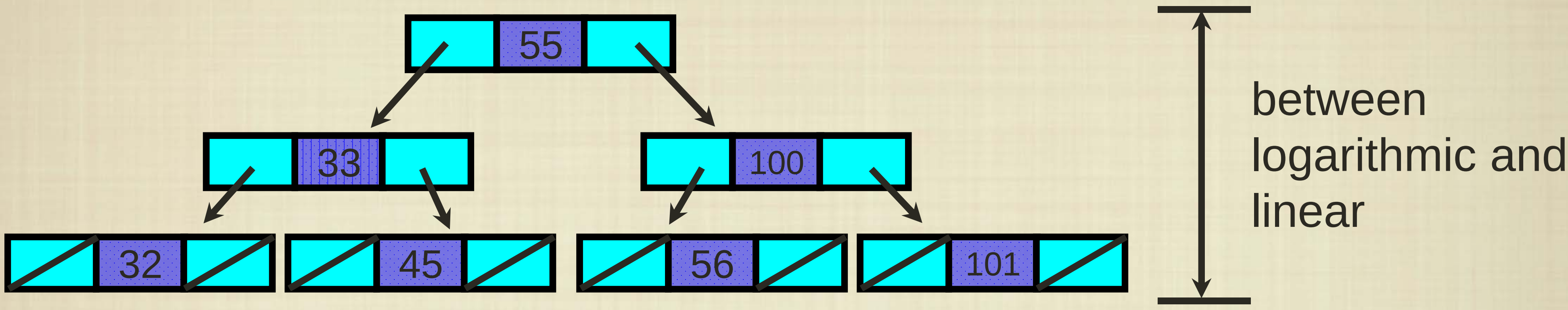


An <u>in-order traversal</u> works by “visiting” the left subtree, then the current item, and then visiting the right subtree.

A <u>pre-order traversal</u> works by “visiting” the item we are at, then the left subtree, and then the right subtree.

Summary of Binary Search Trees



- The time to perform operations in binary search trees is highly dependent on how they are built.
- The best-case height of a binary tree is logarithmic in the number of elements; there are sophisticated techniques (AVL, red-black) for ensuring this height is logarithmic in the number of elements in the worst-case.
- Do tree data structures always have to be binary? Are they always used to add/remove/find elements in a collection?