

# 1. Homework

Due **9/11/13** at the beginning of class

## 1. Hexadecimal numbers (8 points)

Hexadecimal numbers are numbers in base 16. They use the following sixteen digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F.

- Convert  $A2F31_{16}$  to decimal.
- Convert  $4576_{10}$  into hexadecimal.
- Convert  $0001000111100000_2$  to hexadecimal. How can you use the fact that  $16 = 2^4$ ?
- If you convert a 32-bit binary number into hexadecimal, how many hexadecimal digits does it have?

## 2. NOR (6 points)

We have mentioned in class that any Boolean function can be expressed using a combination of  $\wedge$ ,  $\vee$ ,  $\neg$ . In practice, however, it is more efficient to manufacture fewer types of gates.

- (2 points) Show that it suffices to only manufacture gates for the operators  $\vee$ ,  $\neg$ , by showing that  $\wedge$  can be implemented using  $\neg$  and  $\vee$  only.
- (4 points) Consider the NOR operator  $\downarrow$  which is defined using the truth table below.  $x \downarrow y$  is equivalent to  $\neg(x \vee y)$ .

$x$	$y$	$x \downarrow y$
0	0	1
0	1	0
1	0	0
1	1	0

Show that both  $\neg$  and  $\vee$  can be implemented using  $\downarrow$ , i.e., it actually suffices to manufacture NOR gates only.

## 3. Adding three bits (10 points)

In this exercise you will design a circuit for adding three bits  $v, x, y$ , resulting in two outputs  $c$  and  $z$  that represent the addition  $v + x + y$ . This is the same as the full adder that we covered in class. Try to make the circuit depth as small as possible.

- (6 points) Express  $c$  and  $z$  with logical formulas using only  $\wedge$ ,  $\vee$ ,  $\neg$  operators. Use a truth table to show your work.
- (4 points) Draw the corresponding circuit diagram.