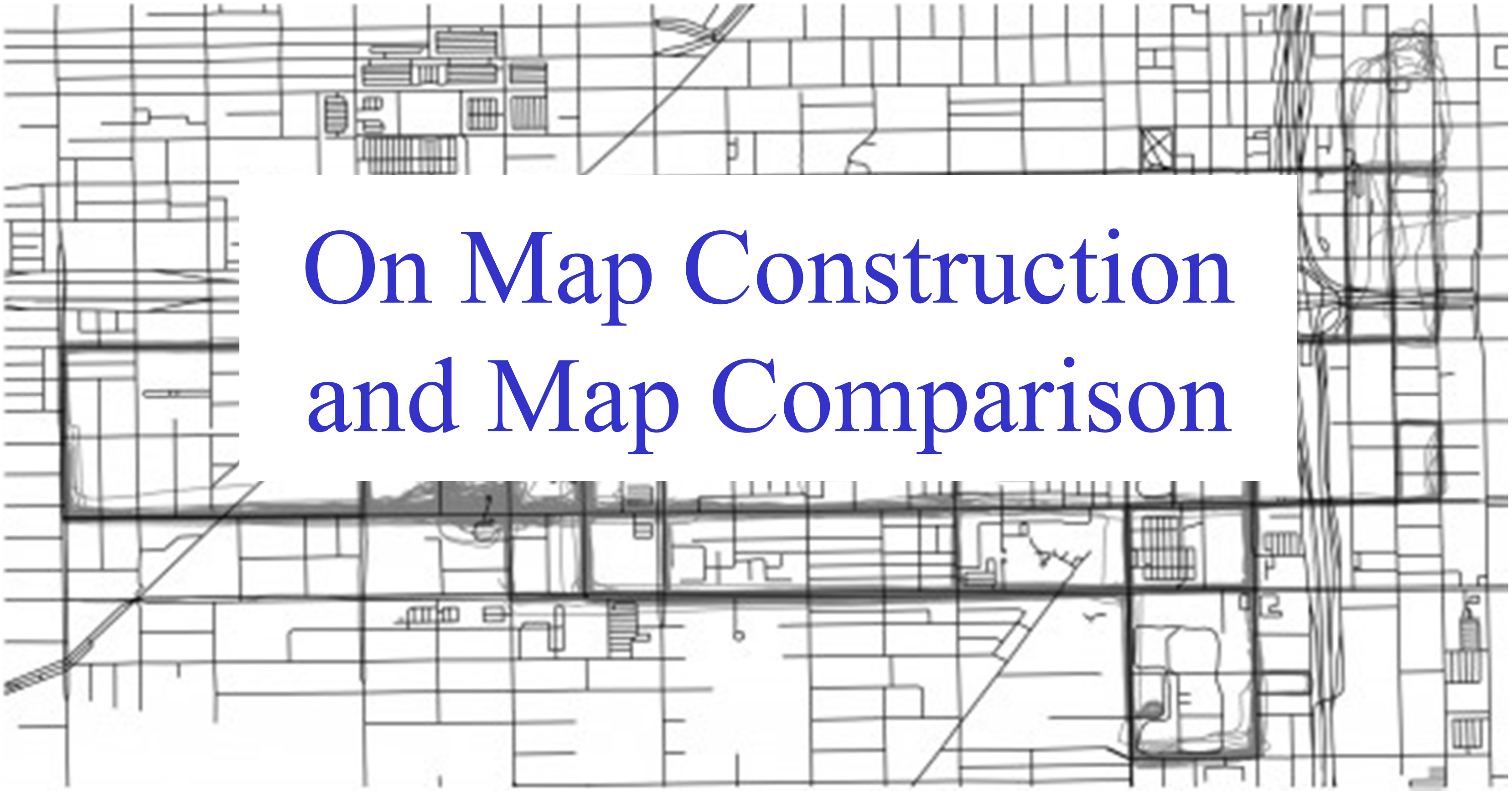


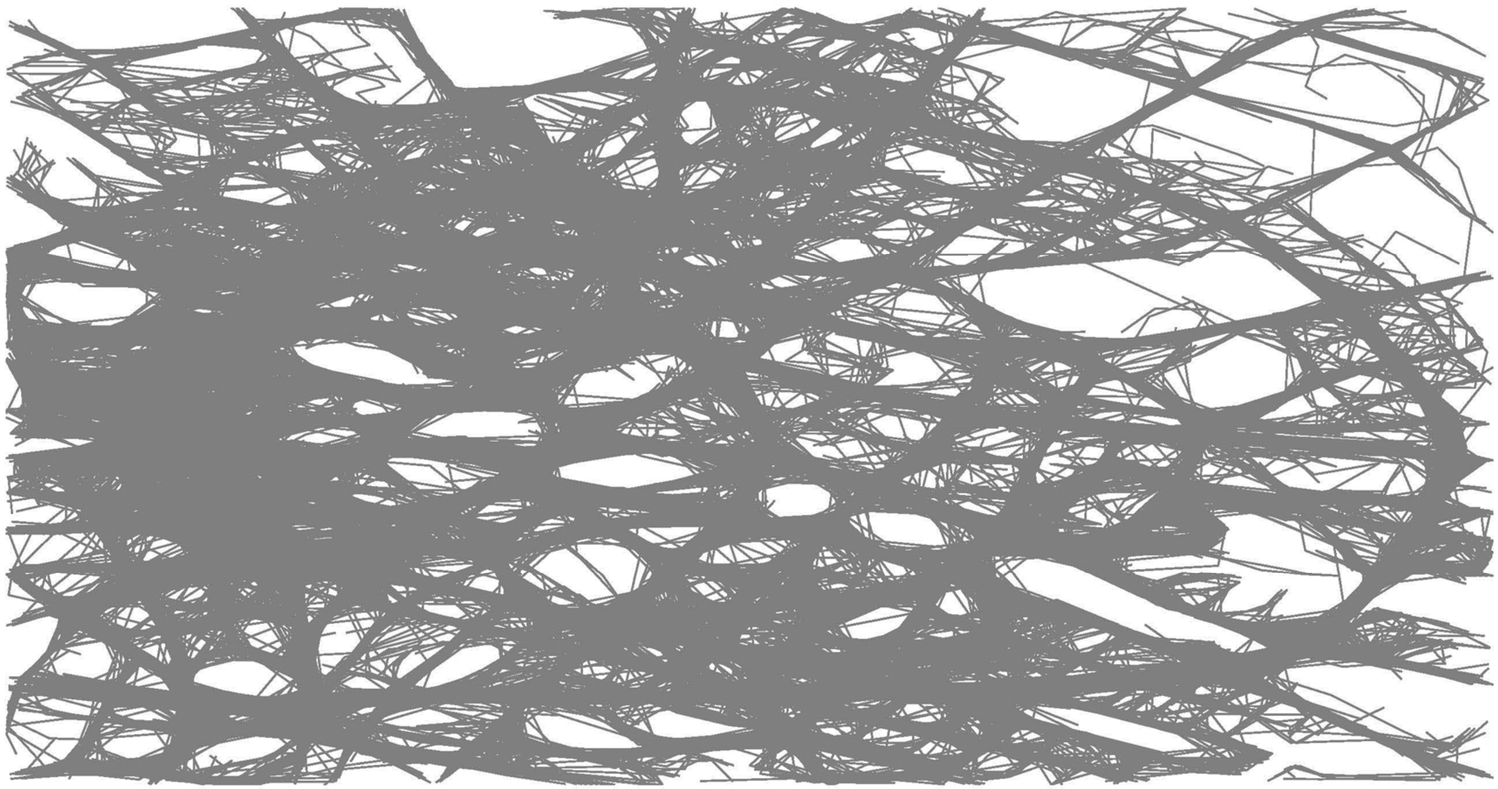
On Map Construction and Map Comparison

Carola Wenk

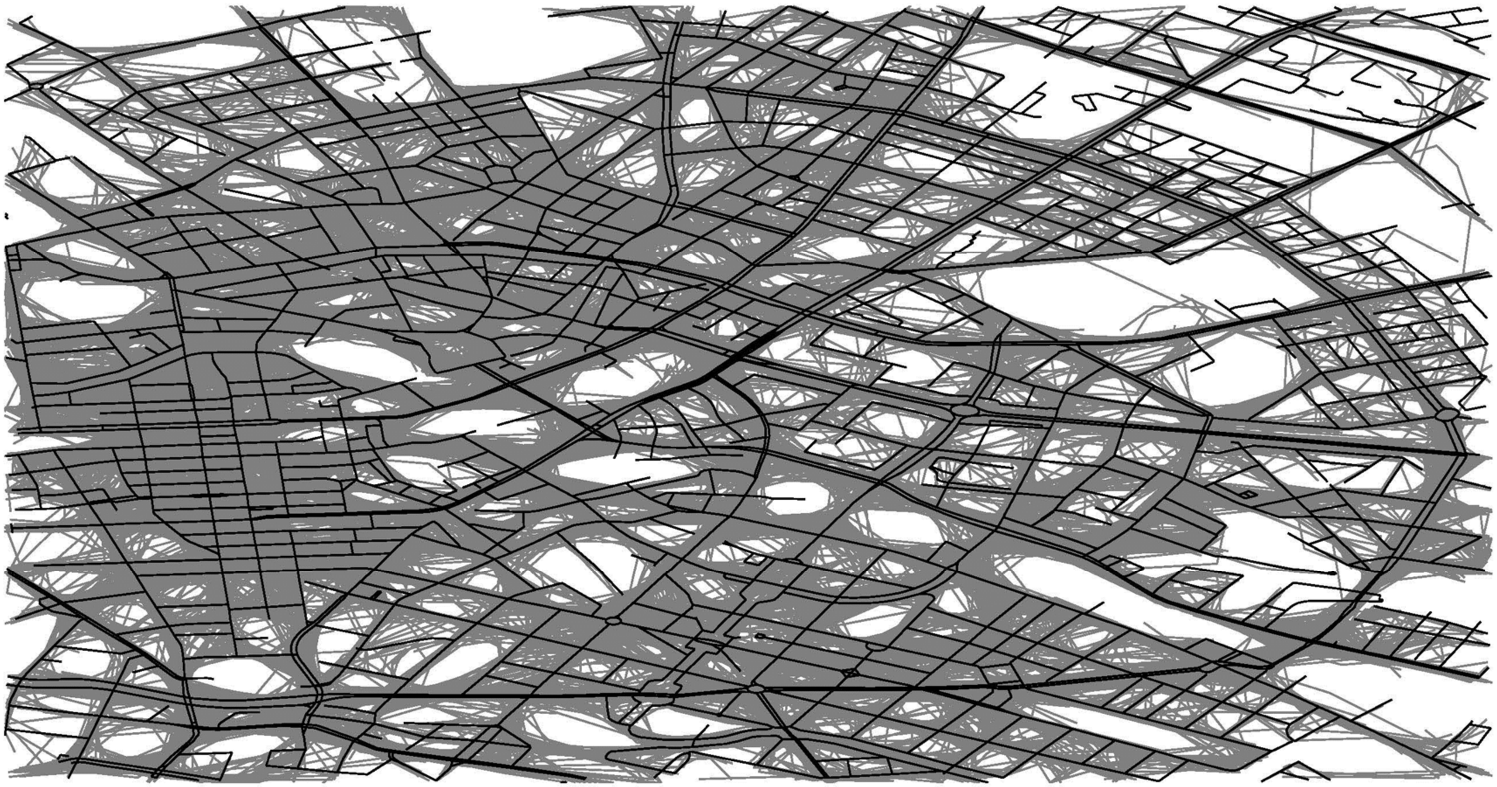
Department of Computer Science

Tulane University

GPS Trajectory Data



GPS Trajectory Data & Roadmap



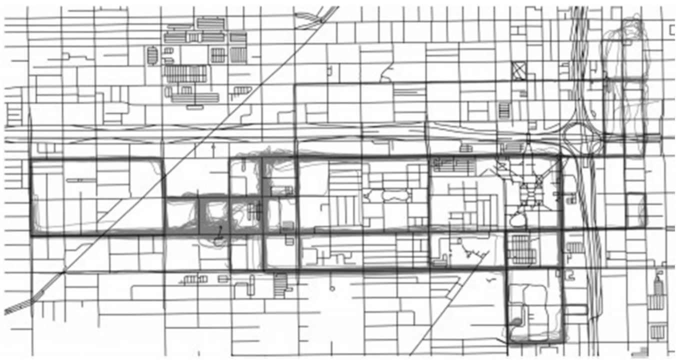
⇒ Map Construction

Which is the Better Roadmap ?

Which is the Better Roadmap ?

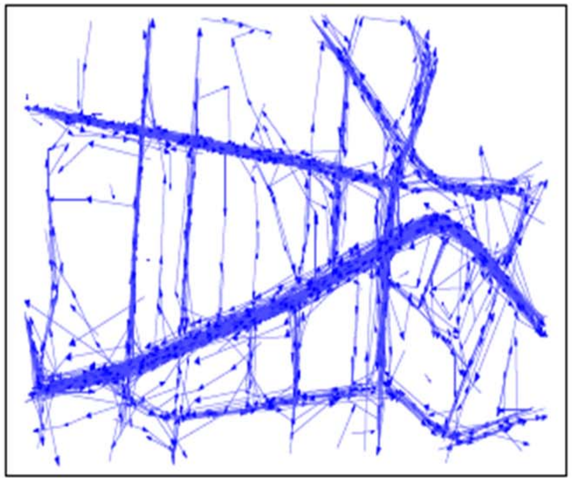
⇒ Map Comparison

Map Construction



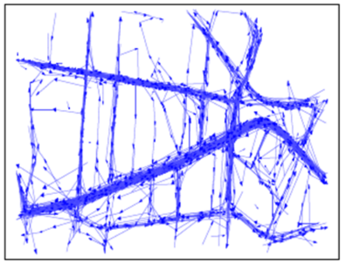
Map Construction

- Given a set of trajectories, compute the underlying road network




- Capturing constrained movement (explicit or implicit streets/routes, animal behavior)
- mapconstruction.org , openstreetmap.org
- Related problem: Map updates

Map Construction



Geometric reconstruction problem:

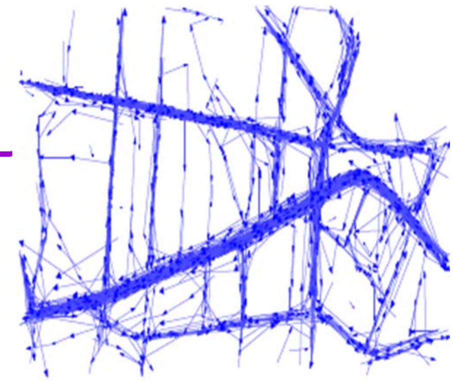
- Given a set of movement-constrained trajectories, extract the underlying geometric graph structure
- Reconstruct a geometric domain that has been sampled with continuous curves that are subject to noise

⇒ Sampling with organized data (trajectories) instead of point clouds

⇒ Need to identify combinatorial information (edges, vertices), as well as geometric representation/embedding

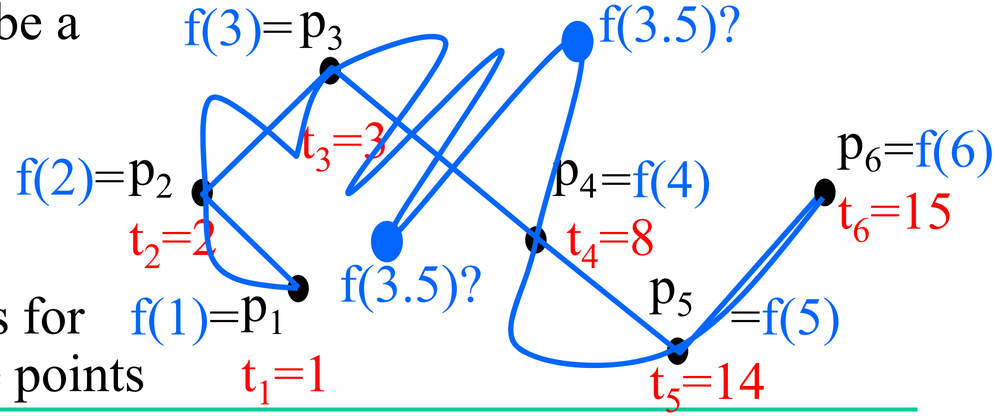
⇒ Clustering & how to represent an edge/street

Trajectories



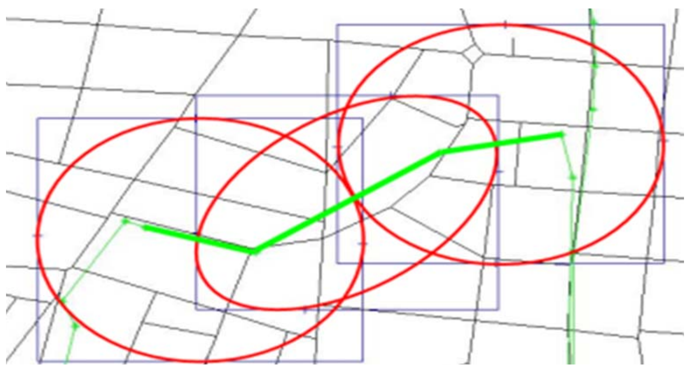
- A **trajectory** is a sequence of position samples: $p_1, \ldots, p_n$
- Each p_i minimally consists of:
  - position measurement (e.g., (x,y)-coordinate)
  - time stamp
  - $\Rightarrow$ e.g., $p_i = (x_i, y_i, t_i)$

- Such a trajectory is a finite sample of a continuous curve $f:[t_1, t_n] \rightarrow R^2$
- For simplicity, f is often assumed to be a piecewise linear interpolation.
- But clearly there are many possible choices for f.
- There are also many possible choices for parameterizations in between sample points



Uncertainty and Error/Noise

- **Measurement error:** Usually modeled as Gaussian noise, or as an error-disk around each measurement point.
- **Sampling error:**
  - Amounts to modeling the transition between two measurements
  - Simple transition model: Linear interpolation.
    Common transition models in ecology: Brownian bridges, Levy walks
  - Simple region-based model: Buffers of fixed radius around each trajectory



⇒ Need **input model**:
E.g., chain of beads model for trajectories

⇒ What is a good **output model**?

[T11] G. Trajcevski, Uncertainty in spatial trajectories, in Y. Zheng, X. Zhou (eds), Computing with Spatial Trajectories: 63-107, 2011.

Map Construction: Some Results

- [CGHS10]: First algorithm with quality guarantees.
  Subsamples trajectories → Dense point cloud.
  Uses local neighborhood simplicial complexes.
  Reconstructs "good" portions of edges.

- [ACCGGM11]: Reconstruct "metric graph" from point cloud.
  Compute almost isometric space with lower complexity.
  Focuses on combinatorial information and not on embedding.
  Quality guarantees assume dense sampling.

- [GSBW11]: Topological approach on neighborhood complex.
  Uses Reeb graph to model skeleton graph (branching structure)

[ACCGGM11] M. Aanjaneya, F. Chazal, D. Chen, M. Glisse, L. Guibas, D. Morozov. Metric graph reconstruction…, SoCG, 2011.
[CGHS10] D. Chen, L. Guibas, J. Hershberger, J. Sun, Road network reconstruction for organizing paths, SODA, 2010.
[GSBW11] X. Ge, I. Safa, M. Belkin, Y. Wang, Data skeletonization via Reeb graphs, Conf. Neural Inf. Proc. Systems: 837-845, 2011.

Map Construction: More Results

- [FK10]: First identify intersections (vertices) using a shape descriptor, then fill in edges.
- [KP12]: Detect intersections from turns and speed change, then fill in edges.
- [BE12]: Kernel Density Estimation based method; pipeline to first create scaffold then map-match trajectories.
- [A**W**12]: Use trajectory information.
  Incrementally add one trajectory after another.
  Use partial Fréchet distance to identify new and existing portions.
  Use min-link algorithm to compute representative curve/edge.

[FK10] A. Fathi, J. Krumm, Detecting road intersections from GPS traces, Geographic Information Science, LNCS 6292: 56-69, 2010.
[KP12] S. Karagiorgou, D. Pfoser, On vehicle-tracking data-based road network generation, 20th ACM SIGPATIAL: 89-98, 2012.
[BE12] J. Biagioni, J. Eriksson, Map inference in the face of noise and disparity, 20th ACM SIGSPATIAL: 79-88, 2012
[A**W**12] M. Ahmed, **C. Wenk**, Constructing Street Networks from GPS Trajectories, ESA: 60-71, 2012.

Map Construction [AW12]

We model the original map and the reconstructed map as embedded undirected graphs in the plane.

We model error associated with each trajectory by a precision parameter ε.

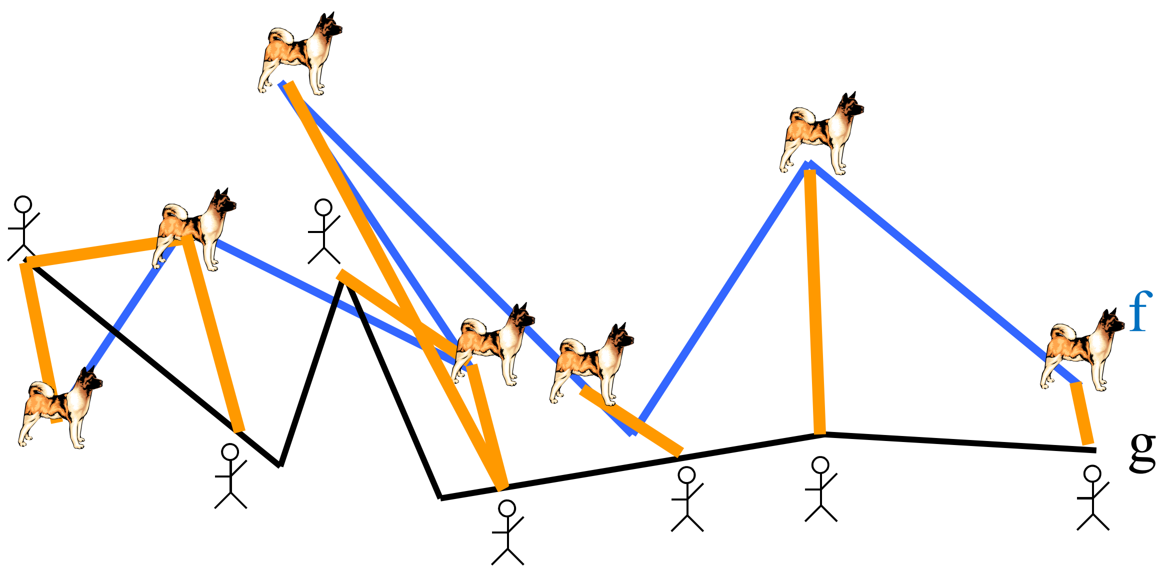
1. We assume each input curve is within Frechet distance ε/2 of a street-path in the original map.
2. (We assume all input curves sample acyclic paths.)
3. Two additional assumptions on original map help us to provide quality guarantees.

[AW12] M. Ahmed, **C. Wenk**, Constructing Street Networks from GPS Trajectories, ESA: 60-71, 2012.

Fréchet Distance for Curves

$$\delta_F(f,g) = \inf_{\alpha,\beta:[0,1]\to[0,1]} \max_{t\in[0,1]} \|f(\alpha(t))-g(\beta(t))\|$$

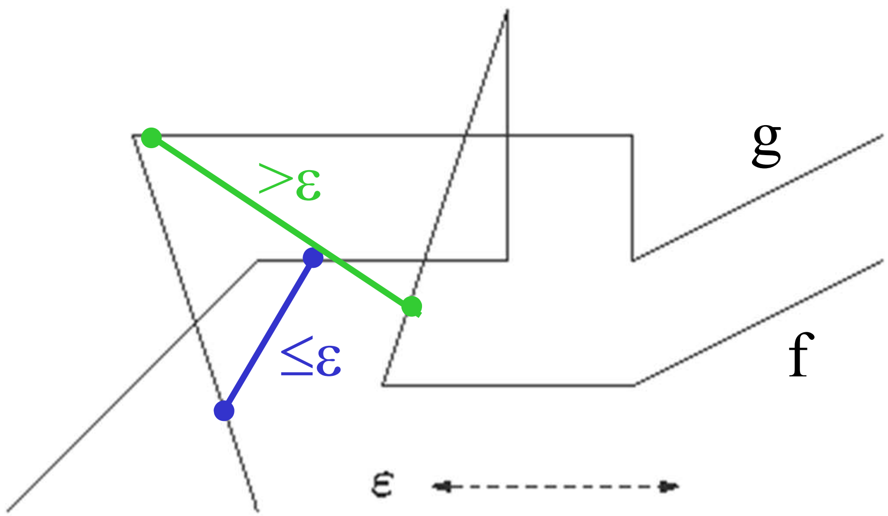
where α and β range over continuous monotone increasing reparameterizations only.

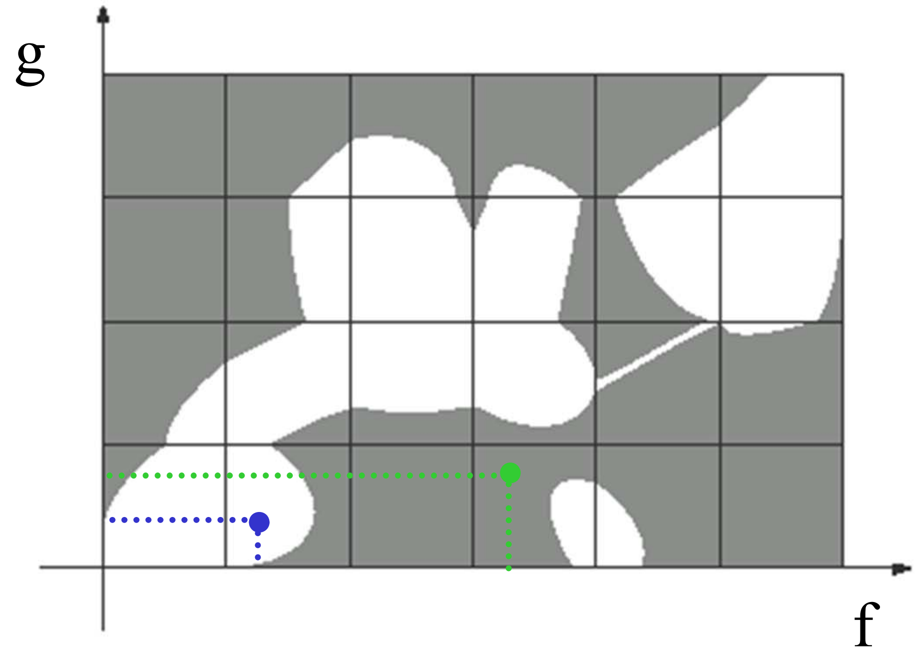


- Man and dog walk on one curve each
- They hold each other at a **leash**
- They are only allowed to go forward
- δ_F is the minimal possible leash length

[F06] M. Fréchet, Sur quelques points de calcul fonctionel, *Rendiconti del Circolo Mathematico di Palermo* 22: 1-74, 1906.

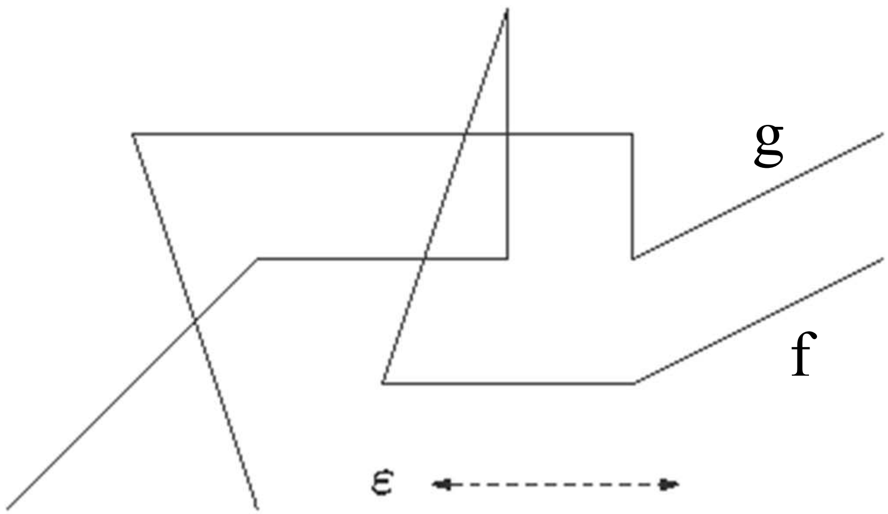
Free Space Diagram

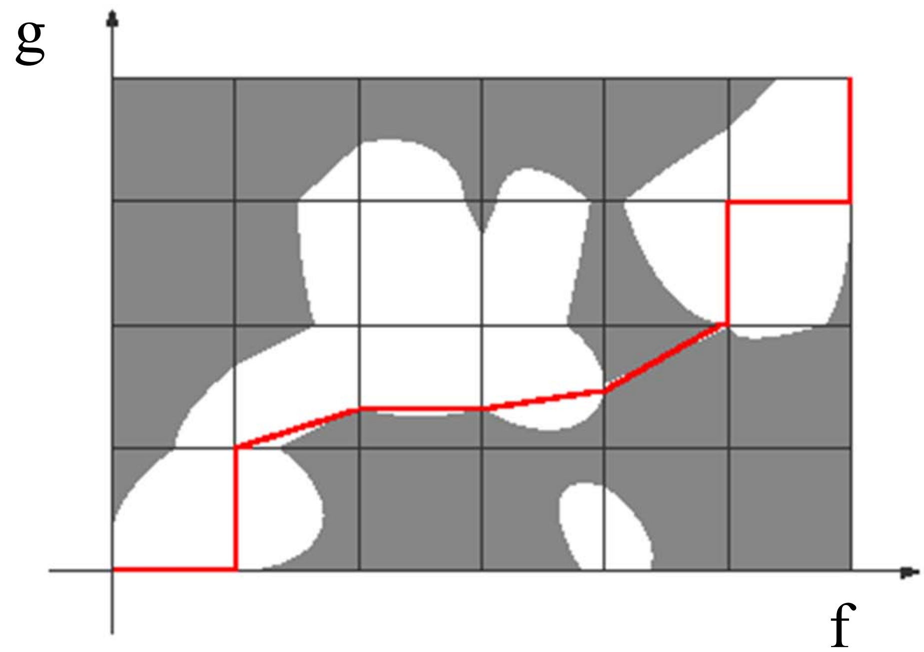




- Let ε > 0 fixed (eventually solve decision problem)
- $F_\varepsilon(f,g) = \{ (s,t) \in [0,1]^2 \mid \| f(s) - g(t) \| \leq \varepsilon \}$ *white points*
  free space of f and g

Free Space Diagram

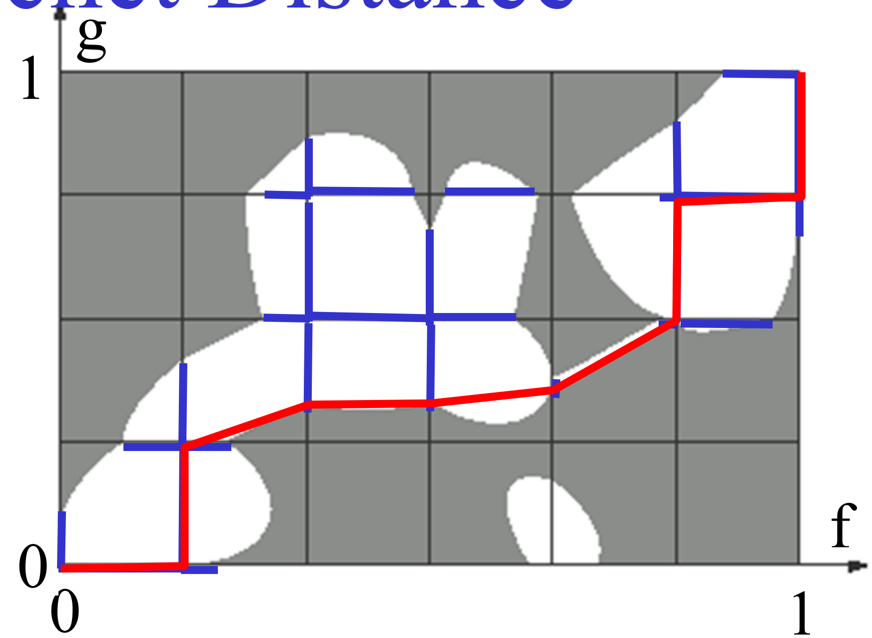




- Monotone path encodes reparametrizations of f and g
- $\delta_F(f,g) \leq \varepsilon$ iff there is a monotone path in the free space from (0,0) to (1,1)
- Such a path can be computed using DP in O(*mn*) time

Compute the Fréchet Distance

- **Solve the decision problem**
  $\delta_F(f,g) \leq \varepsilon$ in O(mn) time:
  - Find monotone path using DP:
  - On each cell boundary compute the interval of all points that are reachable by a monotone path from (0,0)
  - Compute a monotone path by backtracking



- **Solve the optimization problem**
  - In practice in O(mn log b) time with binary search and b-bit precision
  - In O(mn log mn) time [AG95] using parametric search (using Cole's sorting trick)
  - In O(mn $\log^2$mn) expected time [CW09] with randomized red/blue intersections

[AG95] H. Alt, M. Godau, Computing the Fréchet distance between two polygonal curves, *IJCGA* 5: 75-91, 1995.
[CW10] A.F. Cook IV, **C. Wenk**, Geodesic Fréchet Distance Inside a Simple Polygon, ACM TALG 7(1), 19 pages, 2010.

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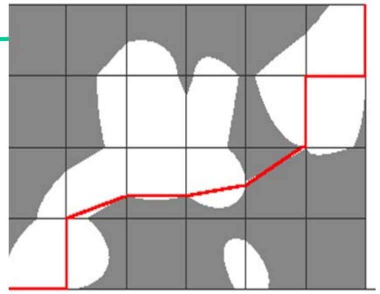
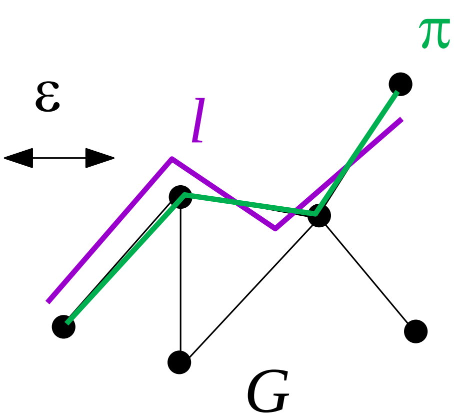
[AW12] M. Ahmed, **C. Wenk**, Constructing Street Networks from GPS Trajectories, ESA: 60-71, 2012.

Subtask: Map Matching



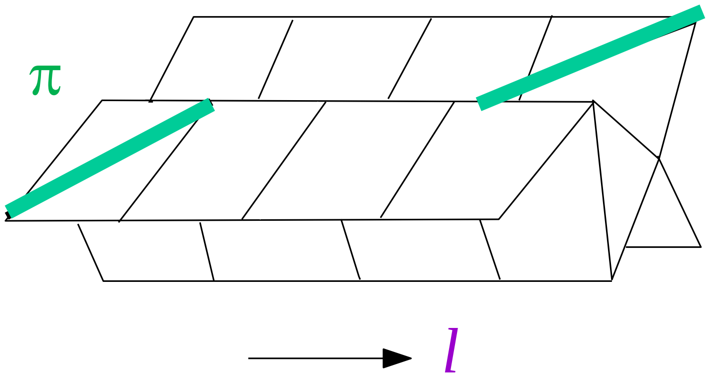
Given: A graph G, a curve l, and a distance parameter ε.

Task: Find a path π in G such that $\delta_F(l,\pi)\leq\varepsilon$



Compute free space surface.

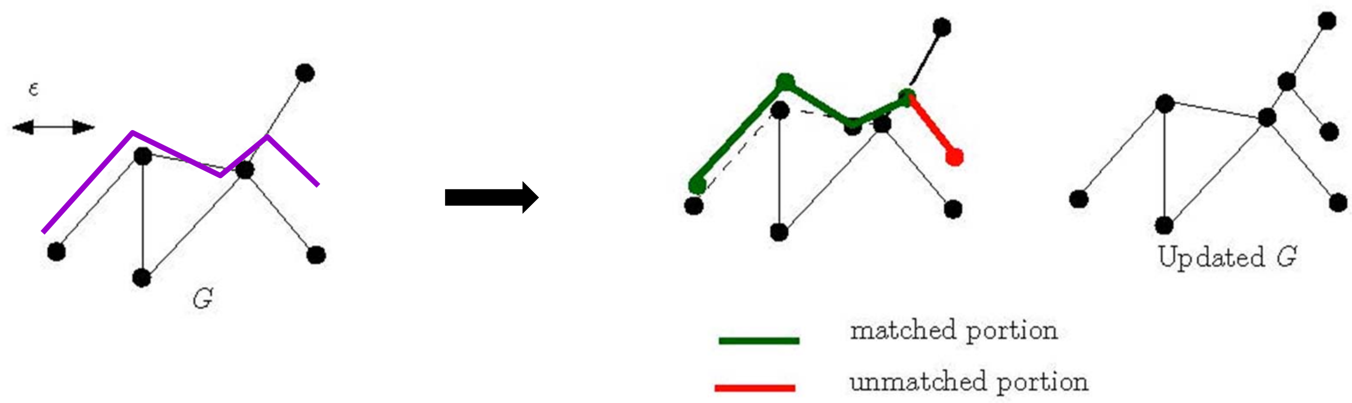
And find path π' in it



Map Construction [AW12]

Incrementally add one trajectory after another.
For each trajectory:

1. Use partial Fréchet distance to identify new and existing portions by combining mapmatching with partial Fréchet distance:



2. Use min-link curve simplification algorithm to reconcile existing portions

[AW12] M. Ahmed, **C. Wenk**, Constructing Street Networks from GPS Trajectories, ESA: 60-71, 2012.

[AW12]

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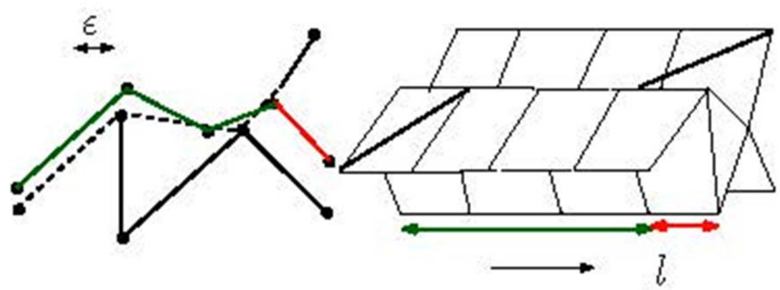
   

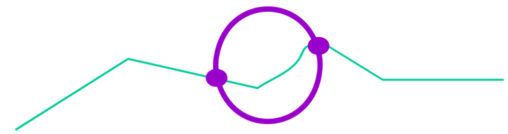
   ⇒ Project free space onto curve:
   white interval = matched portion, black interval = unmatched portion
2. Use min-link curve simplification algorithm to reconcile existing portions

[AW12] M. Ahmed, **C. Wenk**, Constructing Street Networks from GPS Trajectories, ESA: 60-71, 2012.

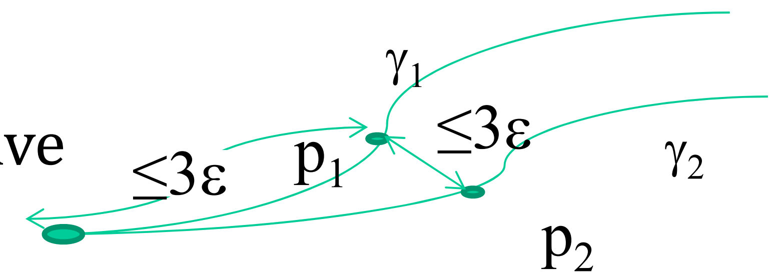
Assumptions

Assumptions on unknown graph:

1. Road fragments are ”good”.
   “good”: Every small circle intersects in just two points



2. Close fragments must have an intersection point

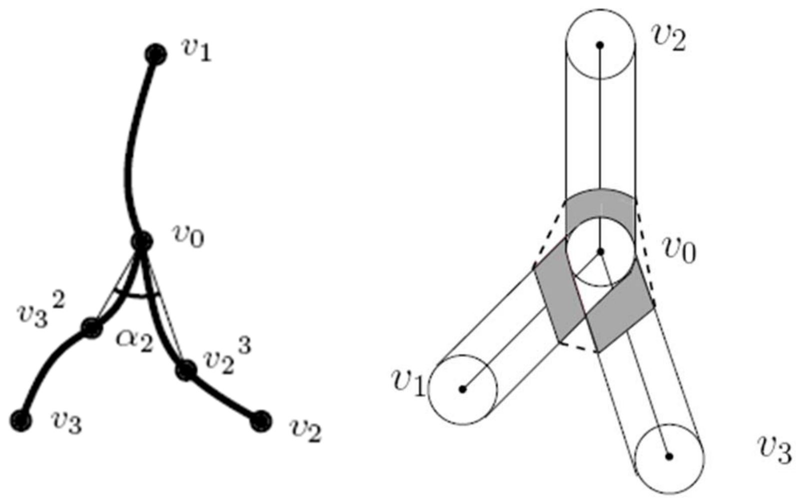


$\Rightarrow$ Projection approach is justified, because free space has special structure. Trajectory can only sample one good section in original network.

[AW12] M. Ahmed, **C. Wenk**, Constructing Street Networks from GPS Trajectories, ESA: 60-71, 2012.

Give quality guarantees

- **Good regions:** We prove the quality guarantee that there is a 1-to-1 correspondence with bounded description complexity between well-separable good portions of original network and reconstructed graph.
- **Bad regions:** We give the first description and analysis of vertex regions.



$\Rightarrow$ It is relatively easy to handle well-sampled clean data.
Deal with noisy data that is not well-sampled and give quality guarantees.

[AW12] M. Ahmed, **C. Wenk**, Constructing Street Networks from GPS Trajectories, ESA: 60-71, 2012.

[AW12]

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   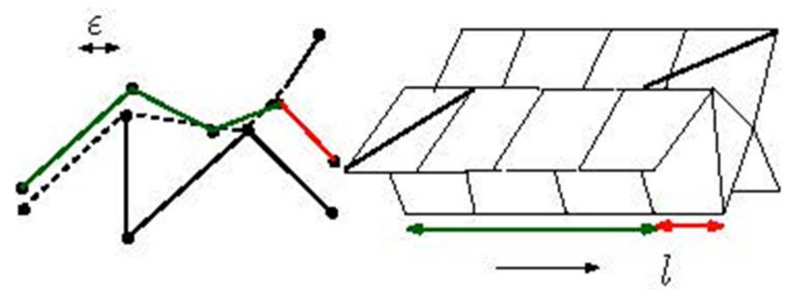

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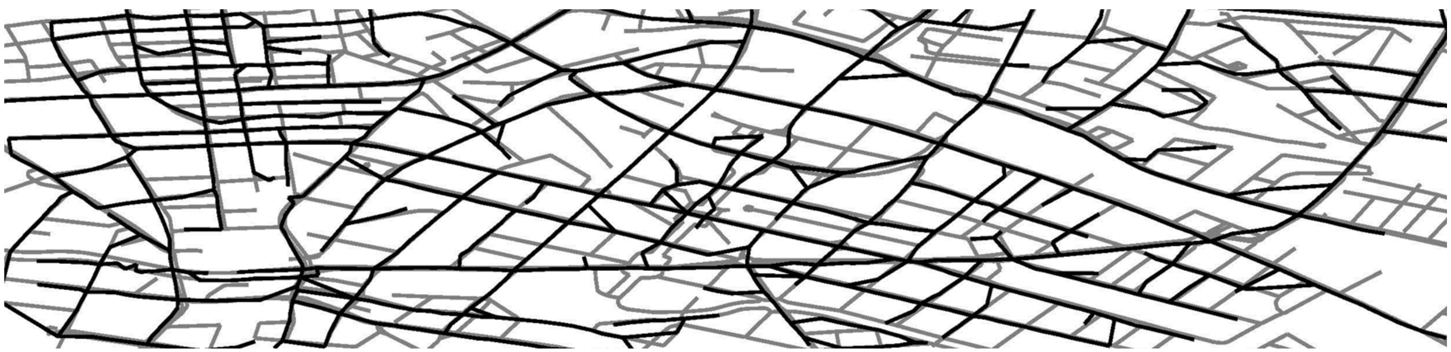
Map Comparison

Compare Constructed Maps

- How can one measure the quality of constructed maps?
- Surprisingly, there is no applicable benchmark map:
  - Professional maps
  - Do not cover the same area and the same details as a given input set of trajectories



- Given two geometric (planar...) graphs embedded in the same plane. How similar are they?
- What if one of the graphs is reconstructed?

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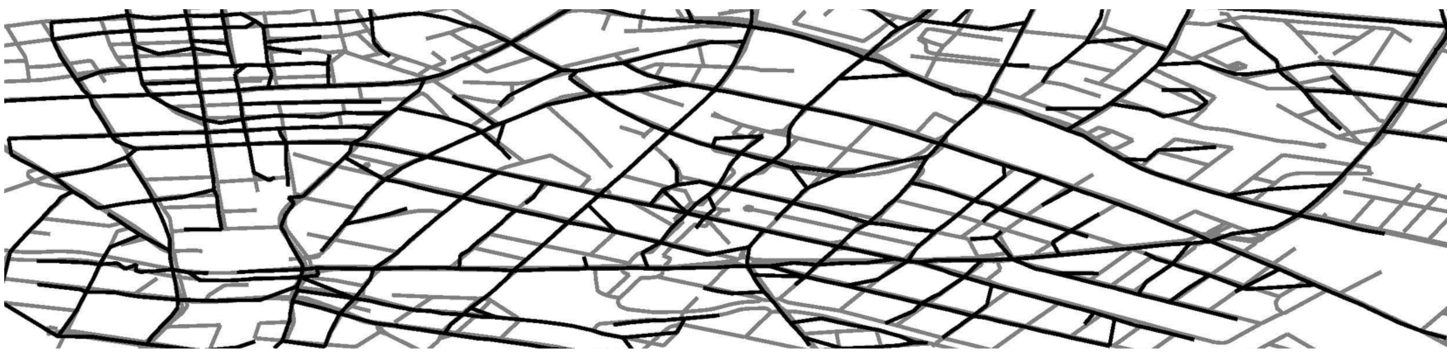

1. Given two geometric (planar...) graphs embedded in the same plane. How similar are they?
2. What if one of the graphs is reconstructed?

Graph Comparison

- Subgraph-isomorphism:
  - Enforces 1-to-1- correspondence, and works on abstract graphs without embedding
- Graph edit distances:
  - Either hard, or only for special graph classes (trees!)
  - Does not incorporate common embedding

⇒ Map comparison is different:

- We have a common embedding
- We need to incorporate partial matching
- 1-to-many assignments may be allowed
- Graphs are planar
- Connectivity should be similar

Distance Measures for Map Comparison

- [BE12b]: Graph sampling-based distance measure in local neighborhood.
  Maximize number of marbles and holes that match 1-to-1.
- [KP12]: Compare shortest paths in both maps, with nearby start and end positions. Ensures similar connectivity/routing properties.
- [BE12], [AKP**W**14]: Overview / benchmark papers

[AFH**W**14]: Considers maps as sets of paths, and compares path sets.

[AF**W**13]: Compares local topology of graphs using persistent homology

[AFH**W**14] M. Ahmed, B. Fasy, K. Hickmann, **C. Wenk**, Path-based distance for street map comparison, arXiv:1309.6131, 2014.
[AF**W**13] M. Ahmed, B. Fasy, **C. Wenk**, Local homology based distance between maps, submitted to SoCG, 2013.

[KP12] S. Karagiorgou, D. Pfoser, On vehicle-tracking data-based road network generation, 20th ACM SIGPATIAL: 89-98, 2012.
[BE12] J. Biagioni, J. Eriksson, Map inference in the face of noise and disparity, 20th ACM SIGSPATIAL: 79-88, 2012
[BE12b] J. Biagioni, J. Eriksson, Inferring road maps from global… TRR: J. of the Transportation Research Board 2291, 61-71, 2012.
[AKP**W**14] M. Ahmed, S. Karagiorgou, D. Pfoser, **C. Wenk**, A Comparison and Evaluation of …, submitted to Geoinformatica, 2014.

[AFHW14] Path-Based Distance

- Directed Hausdorff distance on path-sets:

$$\overrightarrow{d}_{G,H}(\pi_G, \pi_H) = \max_{p_G \in \pi_G} \min_{p_H \in \pi_H} \delta_F(p_G, p_H)$$

- π_G path-set in G, and π_H path-set in H
- We prove that using the set of paths of link-length three approximates the overall distance, if vertices in G are well-separated and have degree $\neq 3$.
- Asymmetry of distance definition is desirable, if G is a reconstructed map and H a ground-truth map.

[AFHW14] M. Ahmed, B. Fasy, K. Hickmann, **C. Wenk**, Path-based distance for street map comparison, arXiv:1309.6131, 2014.
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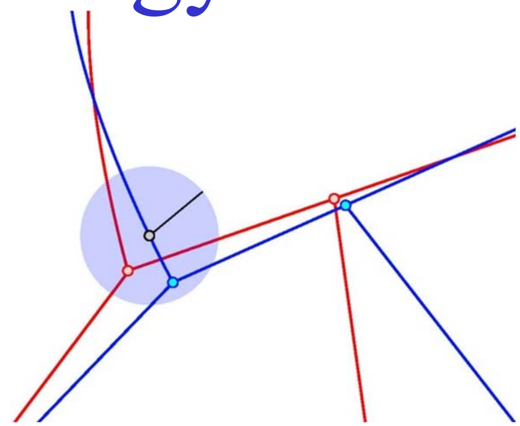
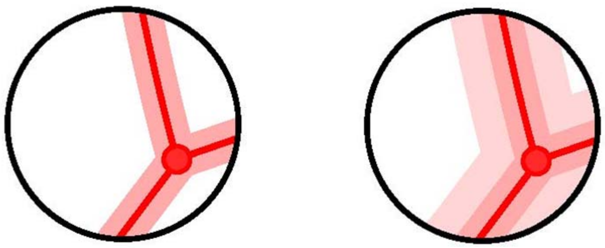
[AKPW14] M. Ahmed, S. Karagiorgou, D. Pfoser, **C. Wenk**, A Comparison and Evaluation of …, submitted to Geoinformatica, 2014.

[AFW13] Local Homology Based Distance

- Consider a common local neighborhood of both maps.



- Consider the cycles of each graph inside this neighborhood.
- Now thicken each graph and track changes in the cycle structure using persistent homology



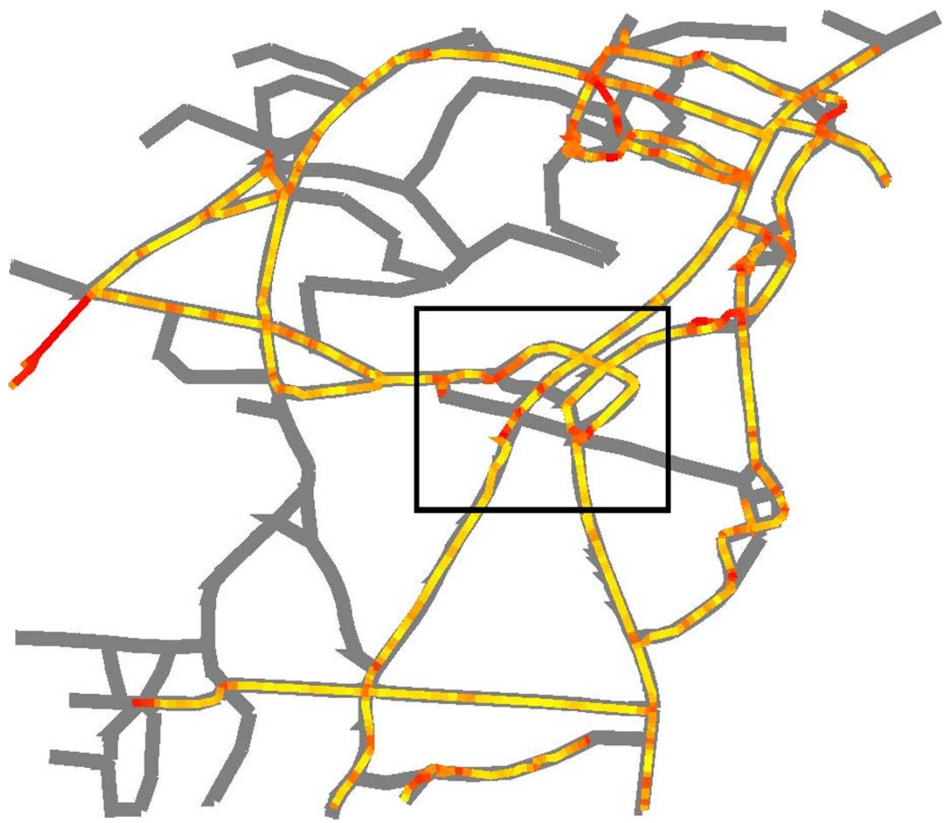
$\Rightarrow$ Use distance between persistence diagrams to compare changing local cycle structure

$\Rightarrow$ Local "signature" that captures local topological similarity of graphs

[AFW13] M. Ahmed, B. Fasy, **C. Wenk**, Local homology based distance between maps, submitted to SoCG, 2013.

[AFW13] Local Homology Based Distance

- Compared two reconstructed maps.
- Local signature captures different topology (missing intersections) well



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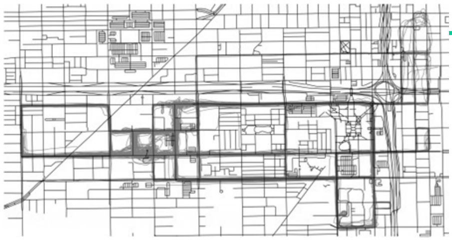
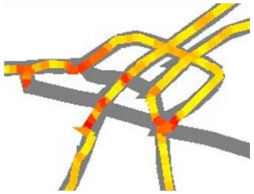


Conclusion



- Map construction and map comparison are recent data-driven problems
- Related to geometric reconstruction, trajectory clustering, shape comparison
- There is a lot of potential for theoretical modeling and algorithms that provide quality guarantees

- Open problems / future work:
  - Map updates
  - More complicated/realistic input and noise models for trajectories
  - More complicated/realistic output models for the maps (vertex regions; directed graphs, with turn information, road categories, etc.)