On Map Construction and Map Comparison

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GPS Trajectory Data
GPS Trajectory Data & Roadmap

⇒ Map Construction
Which is the Better Roadmap?
Which is the Better Roadmap?

⇒ Map Comparison
Map Construction
Map Construction

• Given a set of trajectories, compute the underlying road network
  ![Image of trajectory data and road network]

• Capturing constrained movement (explicit or implicit streets/routes, animal behavior)
  • mapconstruction.org, openstreetmap.org

• Related problem: Map updates
Map Construction

Geometric reconstruction problem:

• Given a set of movement-constrained trajectories, extract the underlying geometric graph structure

• Reconstruct a geometric domain that has been sampled with continuous curves that are subject to noise

⇒ Sampling with organized data (trajectories) instead of point clouds

⇒ Need to identify combinatorial information (edges, vertices), as well as geometric representation/embedding

⇒ Clustering & how to represent an edge/street
A trajectory is a sequence of position samples: $p_1, \ldots, p_n$

- Each $p_i$ minimally consists of:
  - position measurement (e.g., $(x, y)$-coordinate)
  - time stamp
  - $\Rightarrow$ e.g., $p_i = (x_i, y_i, t_i)$

Such a trajectory is a finite sample of a continuous curve $f:[t_1, t_n] \rightarrow \mathbb{R}^2$

For simplicity, $f$ is often assumed to be a piecewise linear interpolation.

But clearly there are many possible choices for $f$.

There are also many possible choices for parameterizations in between sample points.
Uncertainty and Error/Noise

- **Measurement error:** Usually modeled as Gaussian noise, or as an error-disk around each measurement point.

- **Sampling error:**
  - Amounts to modeling the transition between two measurements
  - Simple transition model: Linear interpolation.
    Common transition models in ecology: Brownian bridges, Levy walks
  - Simple region-based model: Buffers of fixed radius around each trajectory

⇒ Need **input model:**
  E.g., chain of beads model for trajectories

⇒ What is a good **output model**?

Map Construction: Some Results


• [ACCGGM11]: Reconstruct “metric graph” from point cloud. Compute almost isometric space with lower complexity. Focuses on combinatorial information and not on embedding. Quality guarantees assume dense sampling.

• [GSBW11]: Topological approach on neighborhood complex. Uses Reeb graph to model skeleton graph (branching structure)

Map Construction: More Results

• [FK10]: First identify intersections (vertices) using a shape descriptor, then fill in edges.

• [KP12]: Detect intersections from turns and speed change, then fill in edges.

• [BE12]: Kernel Density Estimation based method; pipeline to first create scaffold then map-match trajectories.

[AW12]: Use trajectory information. Incrementally add one trajectory after another. Use partial Fréchet distance to identify new and existing portions. Use min-link algorithm to compute representative curve/edge.

Map Construction [AW12]

We model the original map and the reconstructed map as embedded undirected graphs in the plane.

We model error associated with each trajectory by a precision parameter $\varepsilon$.

1. We assume each input curve is within Frechet distance $\varepsilon/2$ of a street-path in the original map.

2. (We assume all input curves sample acyclic paths.)

3. Two additional assumptions on original map help us to provide quality guarantees.

Fréchet Distance for Curves

\[ \delta_F(f,g) = \inf_{\alpha,\beta:[0,1] \to [0,1]} \max_{t \in [0,1]} ||f(\alpha(t)) - g(\beta(t))|| \]

where \( \alpha \) and \( \beta \) range over continuous monotone increasing reparameterizations only.

- Man and dog walk on one curve each
- They hold each other at a leash
- They are only allowed to go forward
- \( \delta_F \) is the minimal possible leash length

Free Space Diagram

- Let $\varepsilon > 0$ fixed (eventually solve decision problem)
- $F_\varepsilon(f,g) = \{ (s,t) \in [0,1]^2 \mid \| f(s) - g(t) \| \leq \varepsilon \}$ white points
  free space of $f$ and $g$
• Monotone path encodes reparametrizations of $f$ and $g$
• $\delta_F(f,g) \leq \varepsilon$ iff there is a monotone path in the free space from $(0,0)$ to $(1,1)$
• Such a path can be computed using DP in $O(mn)$ time
Compute the Fréchet Distance

- **Solve the decision problem**
  \( \delta_F(f,g) \leq \varepsilon \) in \( O(mn) \) time:
  - Find monotone path using DP:
  - On each cell boundary compute the interval of all points that are reachable by a monotone path from \((0,0)\)
  - Compute a monotone path by backtracking

- **Solve the optimization problem**
  - In practice in \( O(mn \log b) \) time with binary search and \( b \)-bit precision
  - In \( O(mn \log mn) \) time [AG95] using parametric search (using Cole’s sorting trick)
  - In \( O(mn \log^2 mn) \) expected time [CW09] with randomized red/blue intersections


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Subtask: Map Matching

Given: A graph $G$, a curve $l$, and a distance parameter $\varepsilon$.

Task: Find a path $\pi$ in $G$ such that $\delta_F(l,\pi) \leq \varepsilon$

Compute free space surface.
And find path $\pi'$ in it.
Map Construction [AW12]

Incrementally add one trajectory after another. For each trajectory:

1. Use partial Fréchet distance to identify new and existing portions by combining mapmatching with partial Fréchet distance:

2. Use min-link curve simplification algorithm to reconcile existing portions

Incrementally add one trajectory after another. For each trajectory:

1. Use partial Fréchet distance to identify new and existing portions by combining mapmatching with partial Fréchet distance:
   
   - Compute free space surface
   - Find path that maximizes matched portion on the curve.

   \[\Rightarrow\] Project free space onto curve:
   white interval = matched portion, black interval = unmatched portion

2. Use min-link curve simplification algorithm to reconcile existing portions

Assumptions

Assumptions on unknown graph:

1. Road fragments are "good". "good": Every small circle intersects in just two points

2. Close fragments must have an intersection point

⇒ Projection approach is justified, because free space has special structure. Trajectory can only sample one good section in original network.

Give quality guarantees

- **Good regions:** We prove the quality guarantee that there is a 1-to-1 correspondence with bounded description complexity between well-separable good portions of original network and reconstructed graph.

- **Bad regions:** We give the first description and analysis of vertex regions.

⇒ It is relatively easy to handle well-sampled clean data. Deal with noisy data that is not well-sampled and give quality guarantees.

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   \[ \Rightarrow \text{Project free space onto curve:} \]
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Map Comparison
Compare Constructed Maps

• How can one measure the quality of constructed maps?
• Surprisingly, there is no applicable benchmark map:
  – Professional maps
  – Do not cover the same area and the same details as a given input set of trajectories

• Given two geometric (planar...) graphs embedded in the same plane. How similar are they?
• What if one of the graphs is reconstructed?
Compare Constructed Maps

• How can one measure the quality of constructed maps?
• Surprisingly, there is no applicable benchmark map:
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Graph Comparison

• Subgraph-isomorphism:
  – Enforces 1-to-1- correspondence, and works on abstract graphs without embedding

• Graph edit distances:
  – Either hard, or only for special graph classes (trees!)
  – Does not incorporate common embedding

⇒ Map comparison is different:
  – We have a common embedding
  – We need to incorporate partial matching
  – 1-to-many assignments may be allowed
  – Graphs are planar
  – Connectivity should be similar
Distance Measures for Map Comparison

- [BE12b]: Graph sampling-based distance measure in local neighborhood.
  Maximize number of marbles and holes that match 1-to-1.

- [KP12]: Compare shortest paths in both maps, with nearby start and end positions. Ensures similar connectivity/routing properties.

- [BE12], [AKPWF14]: Overview / benchmark papers

  - [AFHW14]: Considers maps as sets of paths, and compares path sets.

  - [AFW13]: Compares local topology of graphs using persistent homology


[AFHW14] Path-Based Distance

• Directed Hausdorff distance on path-sets:
  \[ \vec{d}_{G,H}(\pi_G, \pi_H) = \max_{p_G \in \pi_G} \min_{p_H \in \pi_H} \delta_F(p_G, p_H) \]

• \( \pi_G \) path-set in G, and \( \pi_H \) path-set in H

• We prove that using the set of paths of link-length three approximates the overall distance, if vertices in G are well-separated and have degree \( \neq 3 \).

• Asymmetry of distance definition is desirable, if G is a reconstructed map and H a ground-truth map.


[AFW13] Local Homology Based Distance

• Consider a common local neighborhood of both maps.

• Consider the cycles of each graph inside this neighborhood.

• Now thicken each graph and track changes in the cycle structure using persistent homology

⇒ Use distance between persistence diagrams to compare changing local cycle structure

⇒ Local “signature” that captures local topological similarity of graphs

[AFW13] Local Homology Based Distance

- Compared two reconstructed maps.
- Local signature captures different topology (missing intersections) well.

Conclusion

• Map construction and map comparison are recent data-driven problems
• Related to geometric reconstruction, trajectory clustering, shape comparison
• There is a lot of potential for theoretical modeling and algorithms that provide quality guarantees

• Open problems / future work:
  – Map updates
  – More complicated/realistic input and noise models for trajectories
  – More complicated/realistic output models for the maps (vertex regions; directed graphs, with turn information, road categories, etc.)