## Questions marked with *** are required for graduate students only.

## 1. ${ }^{* * *}$ max-min fairness ( 5 points)

Recall the definition of max-min fairness. An allocation $x_{r}^{*}$ is called max-min fair if it satisifes the following property: if there is any other allocation $\left\{x_{r}\right\}$ such that $x_{s}>x_{s}^{*}$ for some user $s$, there must be another user $u$ where $x_{u}<x_{u}^{*} \leq x_{s}^{*}$. Prove that max-min fairness implies that $\min _{r} x_{r}^{*} \geq \min _{r} x_{r}$ for any other allocation $\left\{x_{r}\right\}$.

## 2. ${ }^{* * *}$ NUM in a simple network ( 10 points)



Consider a two-link, three-user network shown in the figure. Assume that the link capacities are $C_{A}=C_{B}=1$. The route of source 0 consists of both links $A$ and $B$; the route of source 1 consists of only link $A$; and the route of source 2 consists of only link $B$. Suppose that the utility functions of the users are given as follows: $U_{0}\left(x_{0}\right)=\log \left(x_{0}\right), U_{1}\left(x_{1}\right)=\log \left(1+x_{1}\right)$, and $U_{2}\left(x_{2}\right)=\log \left(1+x_{2}\right)$. Compute the data transmission rates of the three users, $x_{0}, x_{1}$, and $x_{2}$, which maximize the sum network utility.

## 3. Switching (5 points)



Consider the switch shown above. Suppose that all datagrams have the same fixed length, that the switch operates in a slotted, synchronous manner, and that in one time slot a datagram can be transferred from an input port to an output port. The switch fabric is a crossbar so that at most one datagram can be transferred to a given output port in a time slot, but different output ports can receive datagrams from different input ports in a single time slot. What is the minimal number of time slots needed to transfer the packets shown from input ports to their output ports in the following two cases, respectively? Explain.
(a) packets are served in a first-come-first-served (FCFS) manner
(b) packets can be served in any order you want (i.e., it need not have HOL blocking)

## 4. Longest prefix matching (6 points)

Consider a datgram network using 32 -bit host addresses. Suppose a router has four links, numbered 0 through 3 , and packets are to be forwarded to the link interfaces as follows:

## Destination Address Range Link Interface

$$
\left.\begin{array}{c}
11000000 \begin{array}{c}
\text { 00000000 00000000 } 00000000 \\
\text { through }
\end{array} \\
1100000000111111111111111111111
\end{array}\right) 00 \text { 0 }
$$

11000001100000000000000000000000
through
11000001111111111111111111111111
otherwise
硅

3

Provide a forwarding table that uses longest prefix matching and forwards packets to the correct link interfaces. Try to make the forwarding table as small as possible.

## 5. Subnets (6 points)

Consider a router that interconnects three subnets: Subnet 1, Subnet 2, and Subnet 3. Suppose all of the interfaces in each of these three subnets are required to have the prefix 223.1.17/24. Also suppose that Subnet 1 is required to support at least 60 interfaces, Subnet 2 is required to support at least 90 interfaces, and Subnet 3 is required to support at least 12 interfaces. Provide three network addresses (of the form a.b.c.d/x) that satisify these constraints. Explain your answer.

## 6. Distance-vector routing (8 points)



Consider the following network with 4 routers. The initial costs of all links are given as follows: $c(x, y)=4, c(x, z)=50, c(y, w)=1, c(z, w)=1, c(y, z)=3$. Suppose that poisoned reverse is
used in the distance-vector routing algorithm. Now suppose that the link cost between $x$ and $y$ increases to 60 . Will there be a count-to-infinity problem even if poisoned reverse is used? Justify your answer.

