CMPS 4750/6750 Computer Networks – Spring 20

Homework 2

Due 02/13/20 at the beginning of class

Note: Questions marked with (***) are required for graduate students only.

1. **Statistical Multiplexing and the Chernoff bound (10 points)**

   Consider a link with bandwidth 10 Mbps, which is shared by $n = 500$ data sources. At any time, a source is active with a probability of 0.2, and the sources become active independently.

   (a) On average, how many sources are active at the same time?

   (b) If a data source transmits at a rate of 50kbps when active, what is the overflow probability? (you only need to give a formula for the overflow probability in this step.)

   (c) Use the Chernoff bound to estimate the overflow probability in (b). (***)

2. **The Chernoff bound for *i.i.d.* Bernoulli random variables (10 points)** (***)

   Let $\{X_i\}$ be *i.i.d.* Bernoulli random variable with mean p. Prove that, for $p < x \leq 1$,

$$\Pr\left(\sum_{i=1}^{n} X_i \geq nx\right) \leq e^{-nD(x\|p)},$$

   where $D(x \parallel p) = x \log \frac{x}{p} + (1-x) \log \frac{1-x}{1-p}$ is the KL divergence between two Bernoulli random variables with parameters x and p, respectively.

3. **(Data Centers, Backhoes, and Bugs) (10 points)**

   Data centers alternate between "working" and "down." There are many reasons why data centers can be down, but for the purpose of this problem we mention only two reasons: (i) a backhoe accidentally dug up some cable, or (ii) a software bug crashed the machines. Suppose that a data center that is working today will be down tomorrow due to backhoe reasons with probability $\frac{1}{6}$, but will be down tomorrow due to a software bug with probability $\frac{1}{4}$. A data center that is down today due to backhoe reasons will be up tomorrow with probability 1. A data center that is down today due to a software bug will be up tomorrow with probability $\frac{3}{4}$. Let $X_k \in \{0, 1, 2\}$ denote the state of a data center in time-slot k, where state 0 means the data center is up, state 1 means the data center is down due to backhoe reasons, and state 2 means the data center is down due to a software bug.

   (a) Draw a DTMC for X_k and give its transtition probability matrix.

   (b) Suppose on day 0, a data center is up with probability 2/3, and is down due to backhoe reasons with probability 1/6, and is down due to a software bug with probability 1/6, what is the probability that the data center is up on day 1?

   (c) Show that the DTMC is irrediculbe and aperiodic. (***)

   (d) What fraction of time is the data center working? (***)

4. **(Geo/D/1 queues) (10 points)** (***)

Consider a single-server queue with infinite buffer. In each time slot, half a packet is served. In other words, it takes two time slots to serve a packet. Packets arrive to this queue according to an *i.i.d.* Bernoulli process with parameter λ. This queue is called the Geo/D/1 queue. Let $X(t)$ denote the number of time slots required to serve the packets in the queue in time-slot t (assume that packets are served in the order in which they arrive). Show that $X(t)$ is a DTMC that is irreducible and aperiodic. Clearly identify the conditions under which the Markov chain has a stationary distribution. Compute the stationary distribution of $X(t)$, the average queue length, and the average waiting time in steady-state.

Hint: The queue length in time slot t is equal to $\lceil X(t)/2 \rceil$.