Policy Design under Collusion

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Motivation

Governments sometimes must develop policy in the presence of agents who cooperate with each other (or would like to) despite conflicts of interest

Examples:

- Procurement auctions
- Antitrust
- Organized criminals/drug tracking organizations
- Coalitions fighting terrorism (e.g. Syria)
- Private sector auctions (e.g. Google)

How to design policy that maximizes the government's objectives in the presence of collusion?

Description

Model:

- Government and players play a repeated game (same stage game throughout)
- Government announces its strategy ("policy") prior to start of the game
- Policy effectively specifies a dynamic game (where stage game depends on history)

Collusion: assume players play the player-optimal (strongly symmetric) equilibrium of that dynamic game

Which policy maximizes government payoff, given players play the most collusive equilibrium?

Simple Example

Players play prisoner's dilemma. Government's payoff is directly opposed to players': negative of sum of players' payoffs

Reward (Government) — Player 1 (rows) vs. Player 2 (columns):

	C	D
C	-6,3,3	-4,0,4
D	-4,4,0	-2,1,1

Attack (Government) — Player 1 (rows) vs. Player 2 (columns):

	C	D
C	-4,2,2	-2,-1,3
D	-2,3,-1	0,0,0

If the government uses the simple policy of either "always retreat" or "always attack", $\delta \geq 1/3$ is sufficient to enforce grim trigger

- suppose the government chooses "always attack". For δ = 0.4, the most conclusive (symmetric) equilibrium: "always (C,C)", average payoff 2

Game Model

Player 1, 2, …,n, have actions $a_i \in A_i$, government ("player 0") has action $a_0 \in A_0$. $A = \Pi_{i \geq 0} A_i$

Symmetric stage game with payoffs $u_i(a_0, a_1, \ldots, a_n)$

- $A_1 = A_2 = \cdots = A_n$
- $u_i(a_0, a_i, a'_{-i}) = u_j(a_0, a_j, a'_{-j})$ if $a_i = a_j$ and $a'_{-i} \in A_{-i}$ and $a'_{-j} \in A_{-j}$ are permutations of each other

Public monitoring: players observe the public history of pervious actions and a public correlation device

- H: set of public histories
- strategy: $\sigma_i : H \to A_i$

Discounted average payoff

$$V_i(\sigma) = (1-\delta)[u_i(a^0) + \delta u_i(a^1) + \delta^2 u_i(a^2) + \cdots]$$

Government announces its strategy $\sigma_0 : H \to A_0$ before the start of the game

Given σ_0, σ_{-0} is a subgame perfect Nash Equilibrium (SPNE) if $\sigma_{-0|h}$ specifies a NE for any history $h \in H$. An SPNE is strongly symmetric if $\sigma_i(h) = \sigma_j(h)$ for all i, j

Policy Design Problem

Assumption: Given σ_0, non-government players play "the most conclusive equilibrium," the highest payoff strongly symmetric SPNE

An optimal government policy σ_0 is given by

$$\max_{\sigma_0} V_0(\sigma_0, \sigma_{-0}) \text{ such that } \sigma_{-0} \in C(\sigma_0)$$

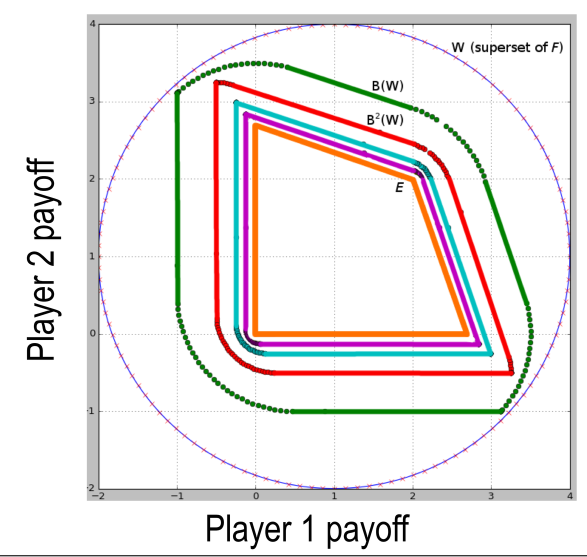
- $V_0(\sigma) = -\sum_{i \geq 1} V_i(\sigma)$
- $C(\sigma_0)$: set of most conclusive equilibrium under σ_0

Related Work

Abreu, Pearce, and Stachetti (1990) (APS) recursive characterization of repeated game equilibrium payoffs as "largest self-generating set"

Computational implementation due to Judd, Yeltekin, and Conklin (2003) (JYC), using polygon approximations of sets at each iteration

	C	D
C	2,2	-1,3
D	3,-1	0,0



Recursive Characterization

An interval $W = [\underline{W}, \overline{W}]$ is (government-) generated by a collection of intervals $\mathbf{W}$ if there exist government action a_0 and continuation payoff sets specified by $\Gamma : A \to \mathbf{W}$ such that for each $w \in W$, there exist a symmetric player action profile a_{-0} and continuation payoffs $\gamma : A \to \mathbb{R}$ such that

- payoffs available: $\gamma(a) \in \Gamma(a)\ \forall a \in A$
- incentive compatibility:

$$w = (1-\delta)u_i(a) + \delta\gamma(a)$$
$$\geq (1-\delta)u_i(a'_i, a_{-i}) + \delta\gamma(a'_i, a_{-i})\ \forall i, a'_i$$

Self-generating Collections

Generating Operator:

- $\mathbf{B}_P(\mathbf{W})$: collection of intervals generated by $\mathbf{W}$
- $\mathbf{B}(\mathbf{W}) = co(\mathbf{B}_P(\mathbf{W}))$
- $\mathbf{W}$ is self-generating if $\mathbf{W} \Subset \mathbf{B}(\mathbf{W})$
  - $\mathbf{W}$ is a contraction of $\mathbf{W}'$ ($\mathbf{W} \Subset \mathbf{W}'$) if for every $W \in \mathbf{W}$, there is $W' \in \mathbf{W}'$ such that $W \subset W'$

❑ **Theorem**: generating operators coverage to equilibrium collection: $\mathrm{F} \Supset \mathrm{B(F)} \Supset \mathrm{B^2(F)} \Supset \cdots \Supset \mathrm{E}$

- $\mathbf{F} = \{F\}$, $F = [\min_{a,i} u_i(a), \max_{a,i} u_i(a)]$ interval of feasible payoffs
- $\mathbf{E}$: set of all strongly symmetric SPNE payoffs

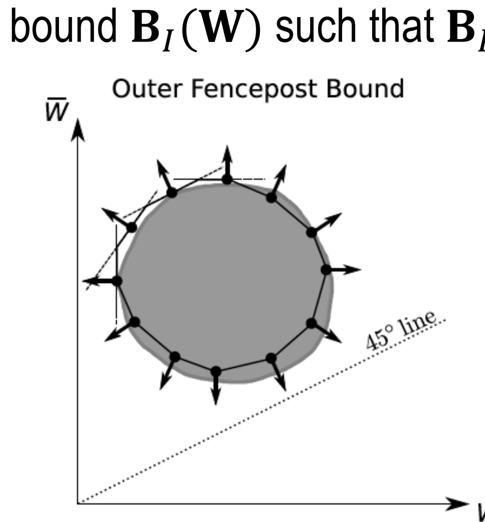
Trace out Optimal Policy

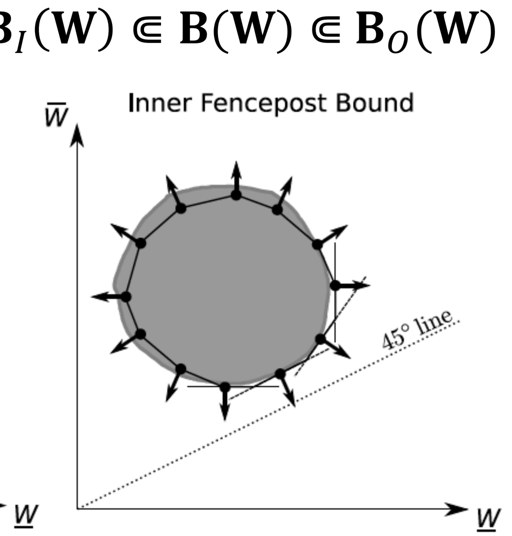
Given $\mathbf{E}$, possible to trace out optimal policy:

1. choose $E \in \mathbf{E}$ that maximizes government payoff
2. find government action a_0 and payoff sets $\Gamma : A \to \mathbf{E}$ that generates E
   - set initial govt policy σ_0(empty history) $= a_0$
3. For each action $a' \in A$, find action a'_0 and payoff sets $\Gamma' : A \to \mathbf{E}$ that generates $\Gamma(a)$
   - set $\sigma_0(a) = a'_0$
4. and so on…

Outer/Inner Bounds

Analogous to JYC, construct outer bound $\mathbf{B}_O(\mathbf{W})$ and inner bound $\mathbf{B}_I(\mathbf{W})$ such that $\mathbf{B}_I(\mathbf{W}) \Subset \mathbf{B}(\mathbf{W}) \Subset \mathbf{B}_O(\mathbf{W})$





Boundary Point Problem

For simplicity, assume two players. Let a_{mn} denote the profile where player 1 plays m-th action and player 2 plays n-th action

To find a boundary point $(\check{\psi}, \hat{\psi})$ in search direction g_q, solve the following optimization problem:

$$\Psi(q) \equiv \sup_{a_0, \Gamma : A \to \mathbf{W}} g_q \cdot \left(\check{\psi}(a_0, \Gamma), \hat{\psi}(a_0, \Gamma)\right)$$
$$s.t.\quad \Gamma(a_{mn}) = \Gamma(a_{nm})\ \forall\, m, n$$

where the inner optimization problems are

$$\check{\psi}(a_0, \Gamma) = \min_{i, \check{w}} (1-\delta)u_1(a_0, a_{ii}) + \delta\check{w}(a_0, a_{ii})$$
$$s.t.\quad \check{w}(a') = \Gamma(a')\ \forall a' \in A$$
$$(1-\delta)u_1(a_0, a_{ii}) + \delta\check{w}(a_0, a_{ii})$$
$$\geq (1-\delta)u_1(a_0, a_{ki}) + \delta\check{w}(a_0, a_{ki})\ \forall k$$

$$\hat{\psi}(a_0, \Gamma) = \max_{i, \hat{w}} (1-\delta)u_1(a_0, a_{ii}) + \delta\hat{w}(a_0, a_{ii})$$
$$s.t. \ldots \text{analogous constraints}\ldots$$

❑ **Theorem**: Let a_0, Γ be given, the optimization problem $\check{\psi}_{ii}(a_0, \Gamma)$ and $\hat{\psi}_{ii}(a_0, \Gamma)$ are feasible iff

$$\max_k \{(1-\delta)u_1(a_0, a_{ki}) + \delta\underline{\Gamma}(a_{ki})\}$$
$$\leq (1-\delta)u_1(a_0, a_{ii}) + \delta\overline{\Gamma}(a_{ii})$$

- MIPs formulated accordingly to solve $\Psi(q)$

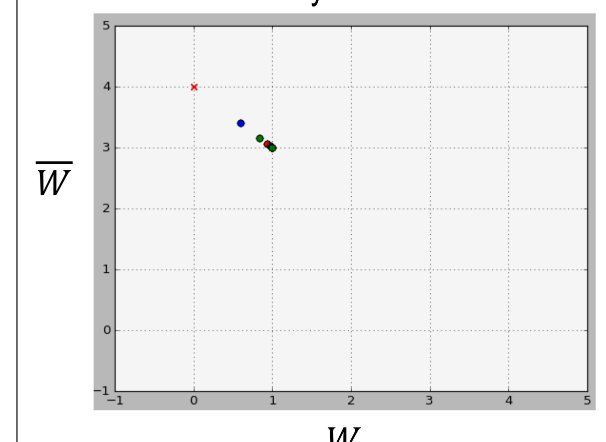
Simple Example (Cont.)

For δ = 0.4, best possible payoff for government is 0, obtained by the following policy:

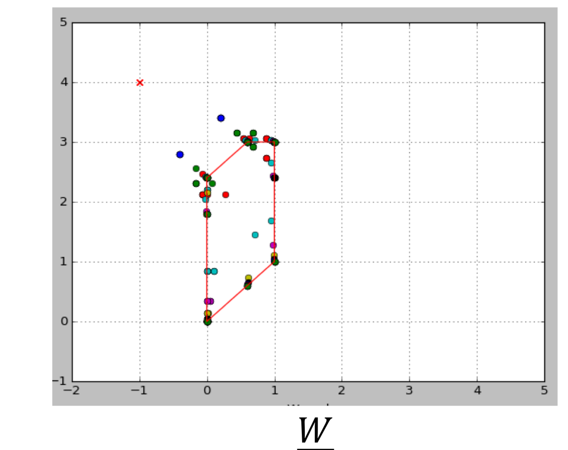
$$\sigma_0(h) = \begin{cases} R & h \text{ contains } (D, C) \text{ or } (C, D) \\ A & \text{otherwise} \end{cases}$$

- Reward players permanently for deviating, otherwise punish
- When not too patient, stops collusion (where simple "always A" policy would fail)
- (Vulnerable to players colluding on asymmetric strategies)

"always R"



full case



Ongoing Work

- Algorithm for tracing out optimal policy
- General government payoff
- Imperfect monitoring